

Amplitudes analysis and exotic states

Alessandro Piloni

FDSA2017, Mexico City, November 7th, 2017

 **Jefferson Lab**

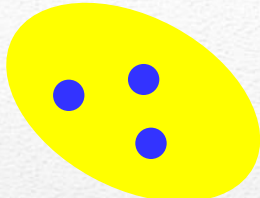


Hadron Spectroscopy

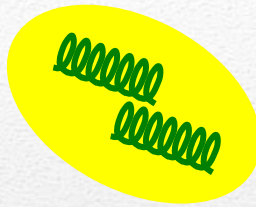
Meson



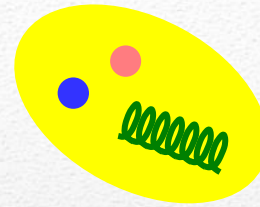
Baryon



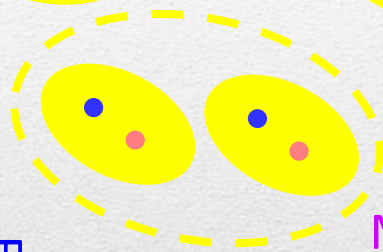
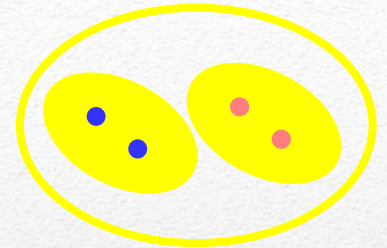
Glueball



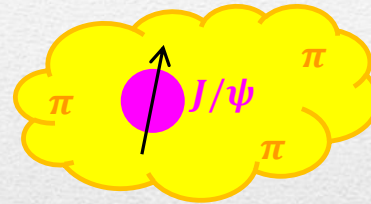
Hybrids



Tetraquark



Molecule



Hadroquarkonium



Experiment

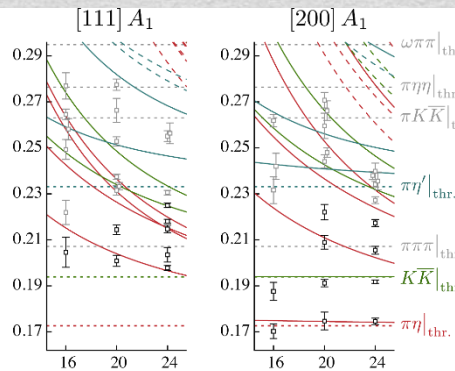
Data



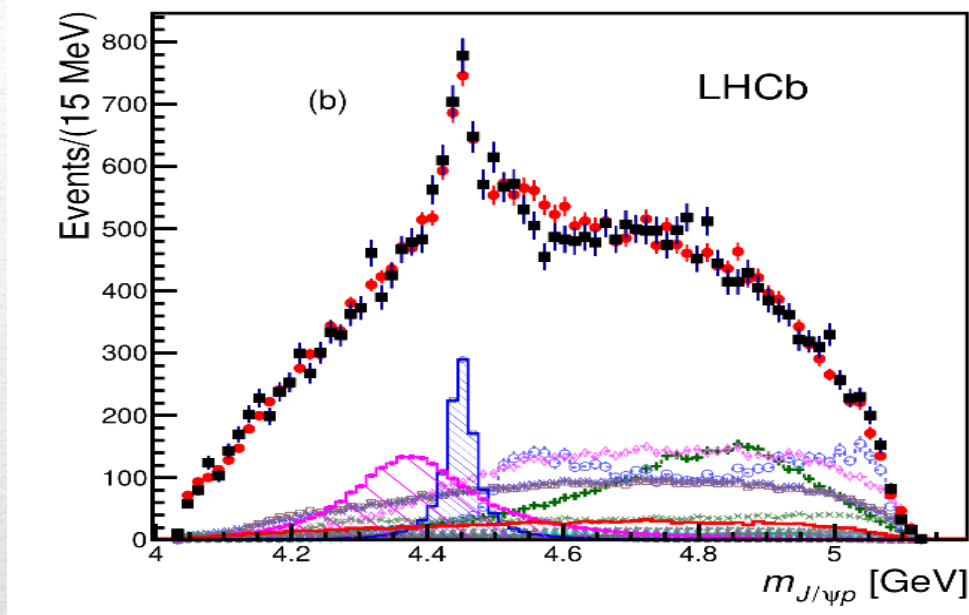
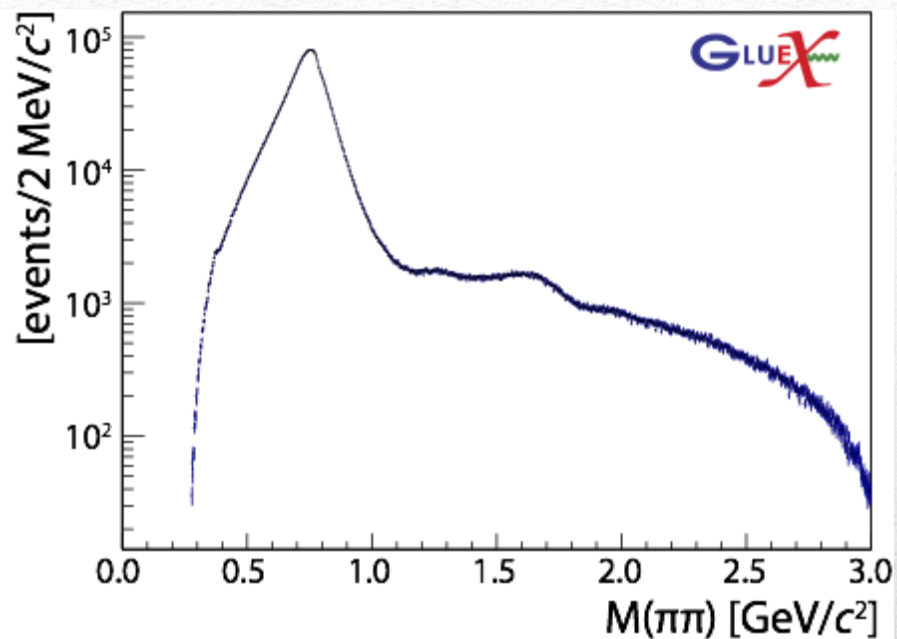
Fundamental properties,
Model building

Lattice QCD

Interpretations on the spectrum leads to understanding fundamental laws of nature



Hadron Spectroscopy

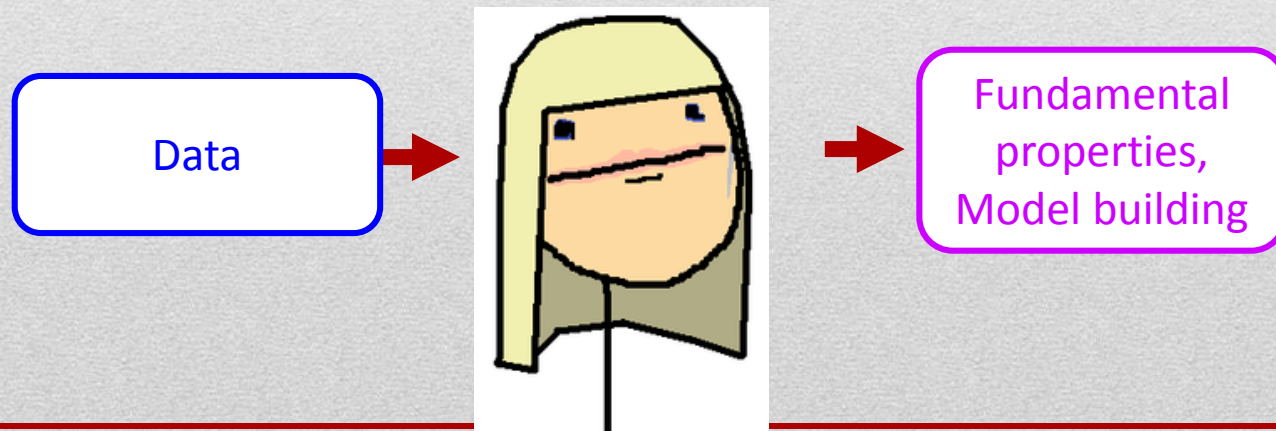
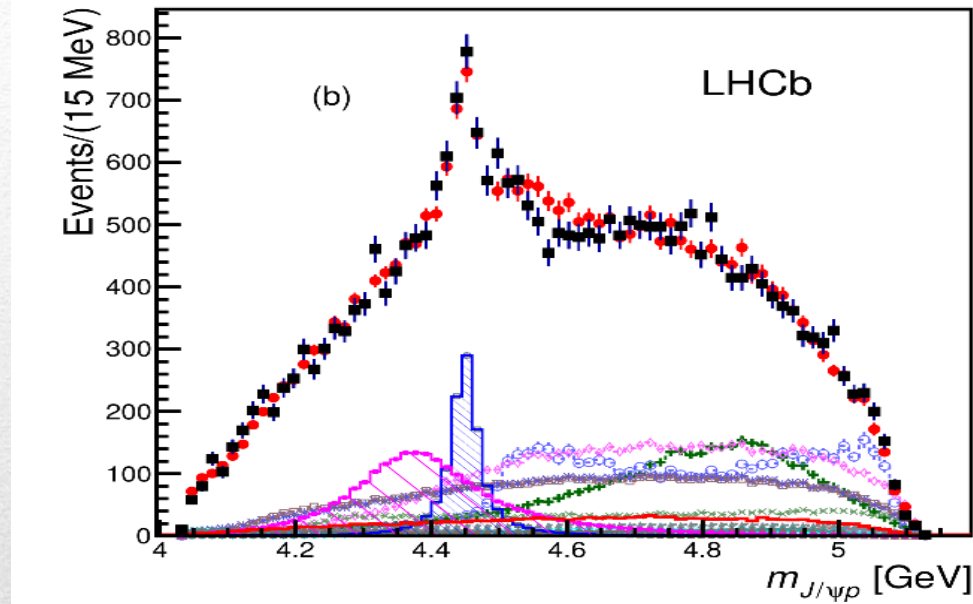
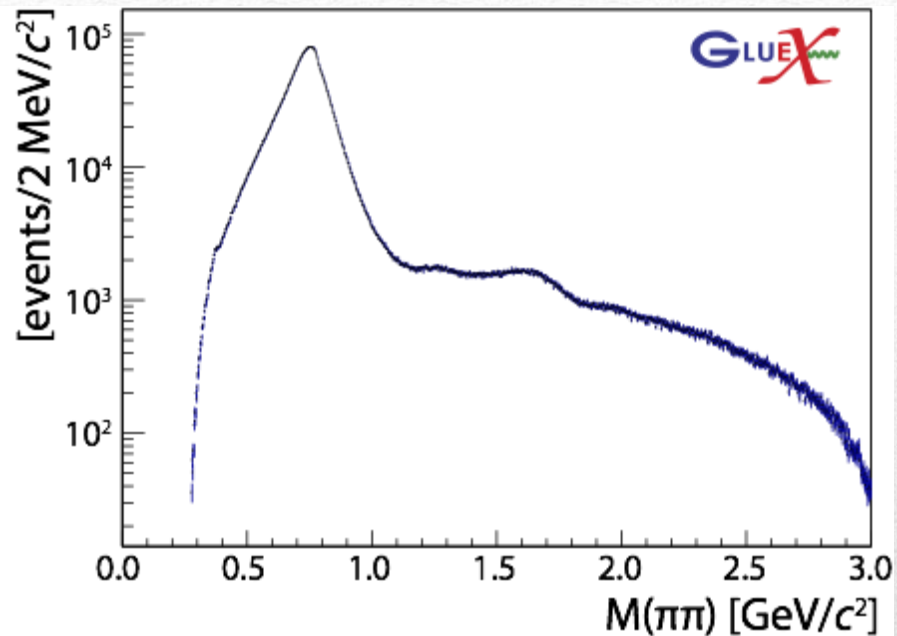


Data

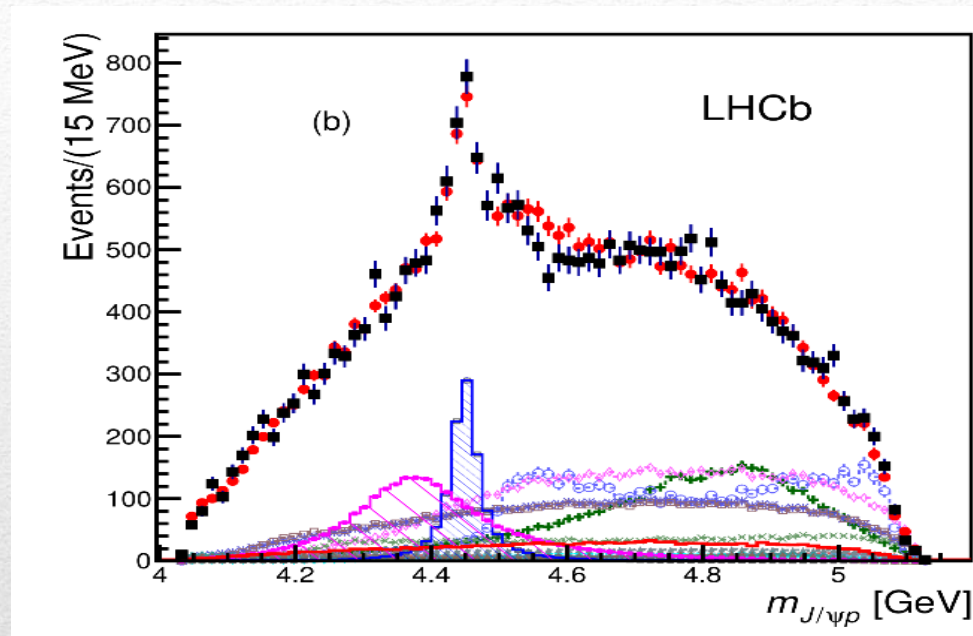
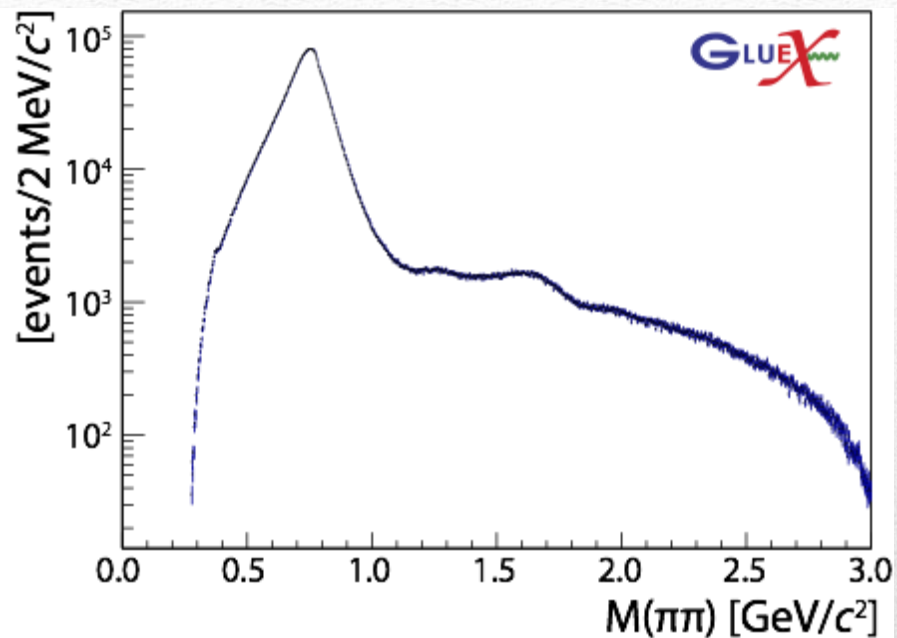


Fundamental
properties,
Model building

Hadron Spectroscopy



Hadron Spectroscopy



Data



Fundamental properties,
Model building

Improvement needed! With great statistics comes great responsibility!

Peter Parker, Ph.D.

Outline

- Recipes to build an amplitude
- The exotic charmonium sector
- Amplitude analysis for the $Z_c(3900)$
- The $X(3872)$, Weinberg and all that
- What's new for the $Y(4260)$
- (Production of exotics at colliders)

Recipes to build an amplitude

M. Mikhasenko, AP et al., to appear

The literature abounds with discussions on the optimal approach to construct the amplitudes for the hadronic reactions

- ▶ Helicity formalism

Jacob, Wick, *Annals Phys.* 7, 404 (1959)

- ▶ Covariant tensor formalisms

Chung, *PRD*48, 1225 (1993)

Chung, Friedrich, *PRD*78, 074027 (2008)

Filippini, Fontana, Rotondi, *PRD*51, 2247 (1995)

Anisovich, Sarantsev, *EPJA*30, 427 (2006)

The common lore is that the former one is nonrelativistic, especially when expressed in terms of LS couplings and the latter takes into account the proper relativistic corrections

How helicity formalism works

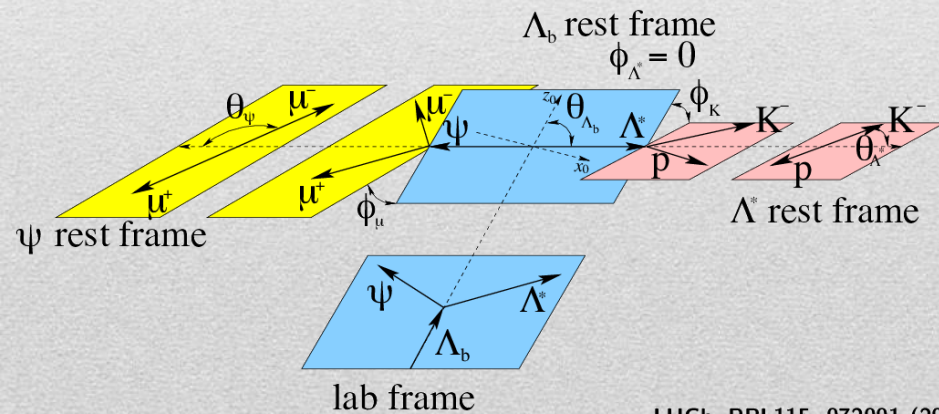
- ▶ Helicity formalism enforces the constraints about rotational invariance
- ▶ It allows us to fix the **angular dependence** of the amplitude
- ▶ What about **energy dependence**?

Example: $\Lambda_b \rightarrow \psi \Lambda^* \rightarrow p K$

$$\mathcal{M}_{\lambda_{\Lambda_b^0}, \lambda_p, \Delta\lambda_\mu}^{\Lambda^*} \equiv \sum_n \sum_{\lambda_{\Lambda^*}} \sum_{\lambda_\psi} \mathcal{H}_{\lambda_{\Lambda^*}, \lambda_\psi}^{\Lambda_b^0 \rightarrow \Lambda_n^* \psi} D_{\lambda_{\Lambda_b^0}, \lambda_{\Lambda^*} - \lambda_\psi}^{\frac{1}{2}}(0, \theta_{\Lambda_b^0}, 0)^*$$

$$\mathcal{H}_{\lambda_p, 0}^{\Lambda_n^* \rightarrow K p} D_{\lambda_{\Lambda^*}, \lambda_p}^{J_{\Lambda_n^*}}(\phi_K, \theta_{\Lambda^*}, 0)^*$$

$$R_{\Lambda_n^*}(m_{Kp}) D_{\lambda_\psi, \Delta\lambda_\mu}^1(\phi_\mu, \theta_\psi, 0)^*,$$



Each set of angles is defined in a different reference frame

LHCb, PRL115, 072001 (2015)

How tensor formalism works

The method is based on the construction of **explicitly covariant** expressions.

- ▶ To describe the decay $a \rightarrow bc$, we first consider the polarization tensor of each particle, $\varepsilon_{\mu_1 \dots \mu_{j_i}}^i(p_i)$
- ▶ We combine the polarizations of b and c into a “total spin” tensor $S_{\mu_1 \dots \mu_S}(\varepsilon_b, \varepsilon_c)$
- ▶ Using the decay momentum, we build a tensor $L_{\mu_1 \dots \mu_L}(p_{bc})$ to represent the orbital angular momentum of the bc system, orthogonal to the total momentum of p_a
- ▶ We contract S and L with the polarization of a

Tensor $\times R_X(m)$ which contain resonances and form factors

What do we know?

- ▶ Energy dependence is not constrained by symmetry
- ▶ Still, there are some known properties one can enforce

$$R_X(m) = B'_{L_{\Lambda_b^0}}(p, p_0, d) \left(\frac{p}{M_{\Lambda_b^0}} \right)^{L_{\Lambda_b^0}}$$

$$BW(m|M_{0X}, \Gamma_{0X}) B'_{L_X}(q, q_0, d) \left(\frac{q}{M_{0X}} \right)^{L_X}$$

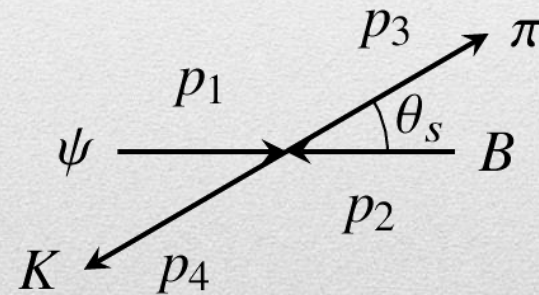
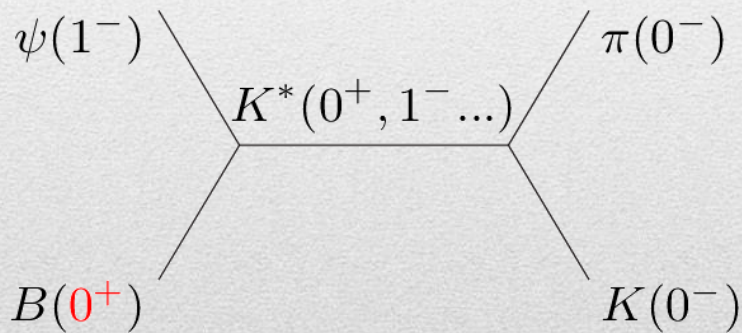
- ▶ **Kinematical singularities**: e.g. barrier factors (known)
- ▶ **Left hand singularities** (need model, e.g. Blatt-Weisskopf)
- ▶ **Right hand singularities** = resonant content (Breit Wigner, K-matrix...)

Kinematics

- ▶ Kinematical singularities appear because of the spin of the **external** particle involved
- ▶ We can write the most general covariant parametrization of the amplitude as
tensor of external polarizations \otimes scalar amplitudes
- ▶ Scalar amplitudes must be **kinematical singularities free**
- ▶ They can be matched to the helicity amplitudes
- ▶ We can get the minimal energy dependent factor
- ▶ Any other additional energy factor would be model-dependent

Example: $B \rightarrow J/\psi \pi K$

To consider the effect of spin, let's consider $B \rightarrow \psi \pi K$
 We focus on the parity violating amplitude for the K^* isobars, scattering kinematics



$$p = \frac{\lambda_{12}^{1/2}}{2\sqrt{s}}, \quad q = \frac{\lambda_{34}^{1/2}}{2\sqrt{s}}, \quad z_s = \frac{s(t-u) + (m_1^2 - m_2^2)(m_3^2 - m_4^2)}{\lambda_{12}^{1/2} \lambda_{34}^{1/2}}$$

Helicity amplitudes

$$A_\lambda = \frac{1}{4\pi} \sum_{j=|\lambda|}^{\infty} (2j+1) A_\lambda^j(s) d_{\lambda 0}^j(z_s)$$

$$d_{\lambda 0}^j(z_s) = \hat{d}_{\lambda 0}^j(z_s) \xi_{\lambda 0}(z_s), \quad \xi_{\lambda 0}(z_s) = \left(\sqrt{1 - z_s^2} \right)^\lambda$$

$\hat{d}_{\lambda 0}^j(z_s)$ is a polynomial of order $j - |\lambda|$

The kinematical singularities of $A_\lambda^j(s)$ can be isolated by writing

$$A_0^j = \frac{m_1}{p\sqrt{s}} (pq)^j \hat{A}_0^j \quad \text{for } j \geq 1,$$

$$A_\pm^j = q (pq)^{j-1} \hat{A}_\pm^j \quad \text{for } j \geq 1,$$

$$A_0^0 = \frac{p\sqrt{s}}{m_1} \hat{A}_0^0 \quad \text{for } j = 0,$$

Identify covariants

Two helicity couplings \rightarrow two independent covariant structures

Important: we are not imposing any intermediate isobar

$$A_\lambda(s, t) = \varepsilon_\mu(\lambda, p_1) \left[(p_3 - p_4)^\mu - \frac{m_3^2 - m_4^2}{s} (p_3 + p_4)^\mu \right] C(s, t) \\ + \varepsilon_\mu(\lambda, p_1) (p_3 + p_4)^\mu B(s, t)$$

$$C(s, t) = \frac{1}{4\pi\sqrt{2}} \sum_{j>0} (2j+1)(pq)^{j-1} \hat{A}_+^j(s) \hat{d}_{10}^j(z_s)$$

$$B(s, t) = \frac{1}{4\pi} \hat{A}_0^0 + \frac{1}{4\pi} \frac{4m_1^2}{\lambda_{12}} \sum_{j>0} (2j+1)(pq)^j \left[\hat{A}_0^j(s) \hat{d}_{00}^j(z_s) + \frac{s + m_1^2 - m_2^2}{\sqrt{2}m_1^2} \hat{A}_+^j(s) z_s \hat{d}_{10}^j(z_s) \right]$$

Everything looks fine **but** the λ_{12} in the denominator

The brackets must vanish at $\lambda_{12} = 0 \Rightarrow s = s_\pm = (m_1 \pm m_2)^2$,

\hat{A}_+^j and \hat{A}_0^j cannot be independent

General expression and comparison

$$\hat{A}_+^j = \langle j-1, 0; 1, 1 | j, 1 \rangle g_j(s) + f_j(s)$$

$$\hat{A}_0^j = \langle j-1, 0; 1, 0 | j, 0 \rangle \frac{s + m_1^2 - m_2^2}{2m_1^2} g_j'(s) + f_j'(s)$$

$$g_j(s_{\pm}) = g_j'(s_{\pm}), \text{ and } f_j(s), f_j'(s) \sim O(s - s_{\pm})$$

All these four functions are **free of kinematic singularity**.

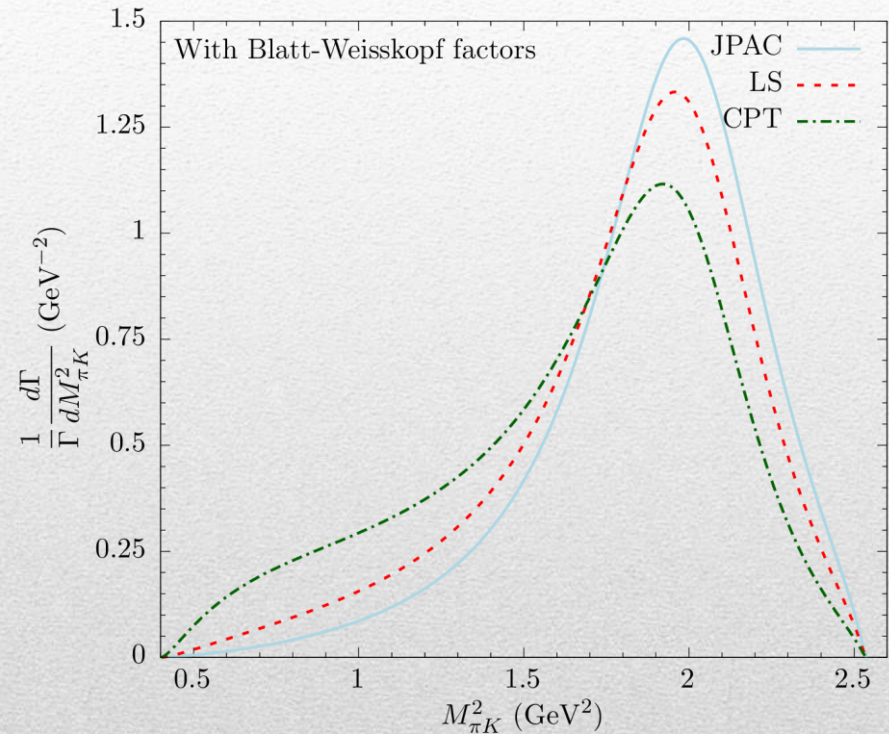
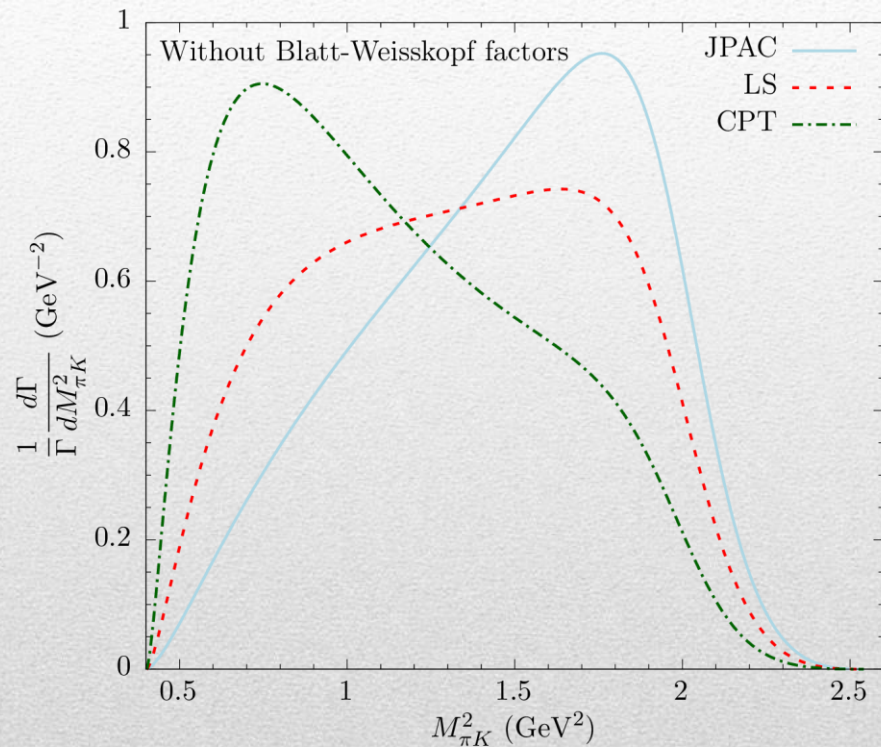
Comparison with tensor formalisms ($j = 1$)

$$g_1 = g_1' = \frac{4\pi}{3} g_S, \quad f_1 = \frac{2\pi\lambda_{12}}{3s} g_D, \quad f_1' = -\frac{4\pi\lambda_{12}}{3s} \frac{s + m_1^2 - m_2^2}{m_1^2} g_D.$$

If the g_S, g_D are the usual Breit-Wigner, the g', f' are fine
but for singularities at $s = 0$

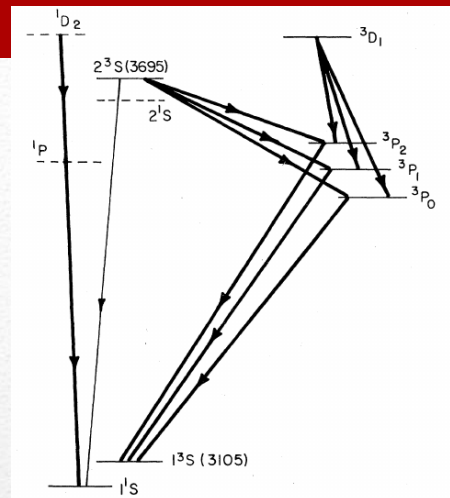
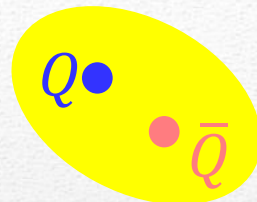
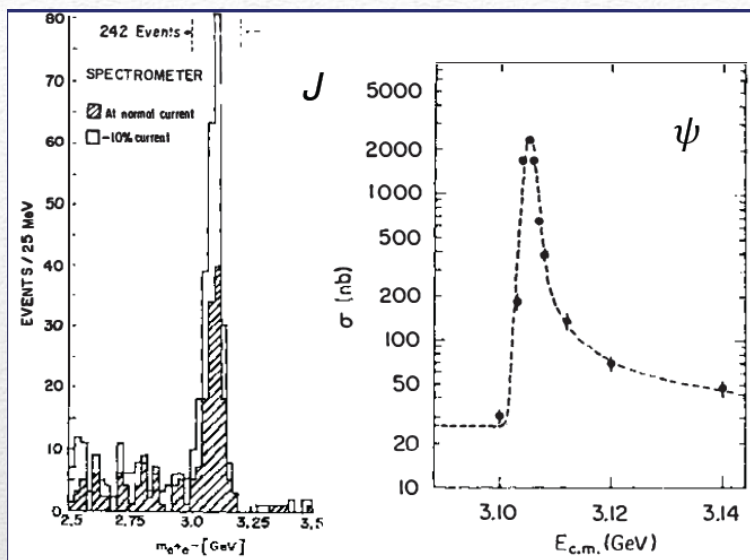
There is no unique recipe to build the right amplitude, but one can ensure the right singularities to be respected

General expression and comparison



The differences can be relevant, in particular if one remove the Blatt-Weisskopf barrier factors

Quarkonium orthodoxy



Potential models

(meaningful when $M_Q \rightarrow \infty$)

$$V(r) = -\frac{C_F \alpha_s}{r} + \sigma r$$

(Cornell potential)

Solve NR Schrödinger eq. \rightarrow spectrum

Effective theories

(HQET, NRQCD, pNRQCD...)

Integrate out heavy DOF



(spectrum), decay & production rates

$$\alpha_s(M_Q) \sim 0.3$$

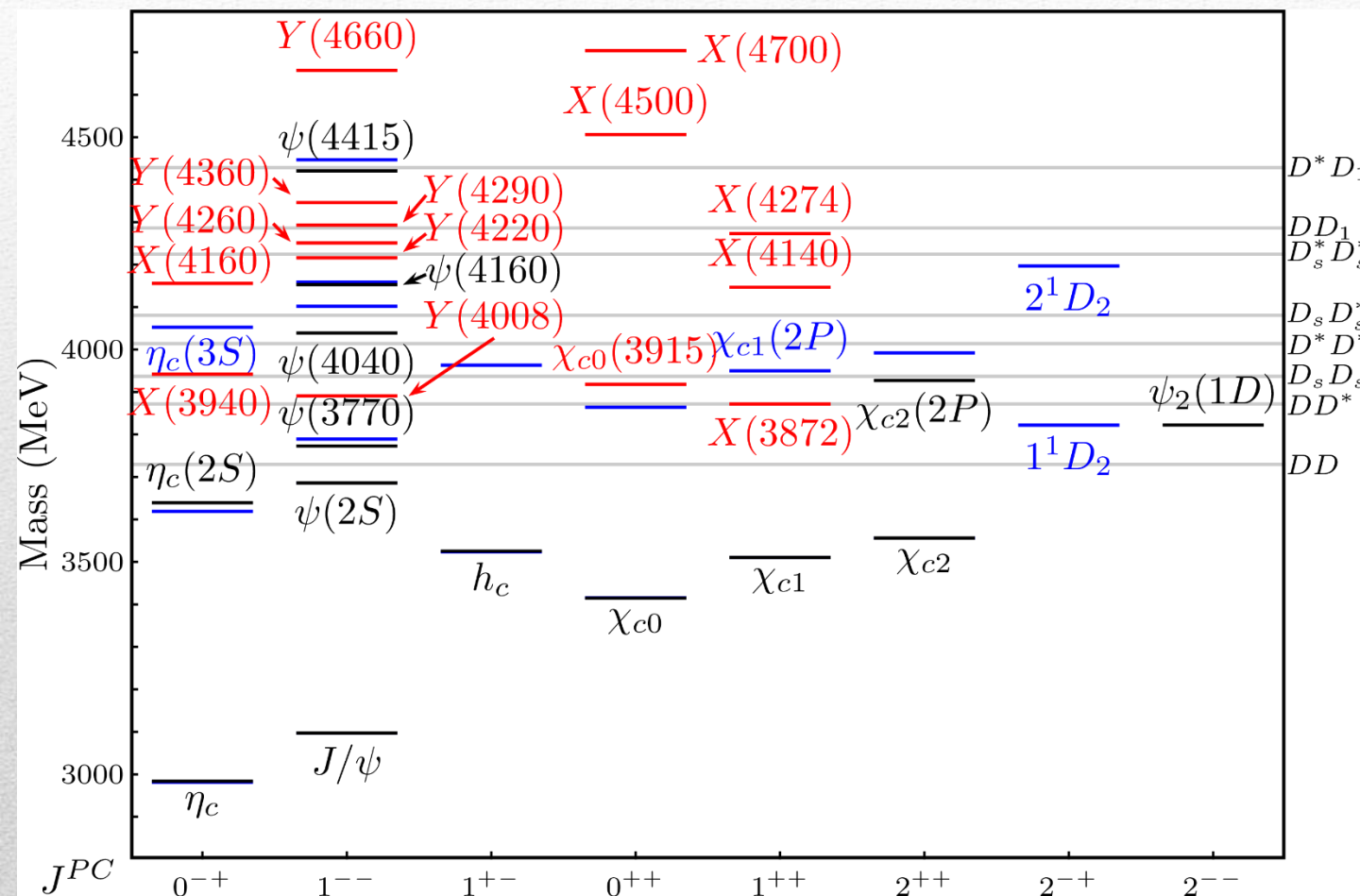
(perturbative regime)

OZI-rule, QCD multipole

Heavy quark spin flip suppressed by quark mass,
approximate heavy quark spin symmetry (HQSS)

Exotic landscape

Esposito, AP, Polosa, Phys.Rept. 668



A host of **unexpected resonances** have appeared

decaying mostly into charmonium + light

Hardly reconciled with usual charmonium interpretation

Charged Z states: $Z_c(3900)$, $Z_c(4020)$

Charged quarkonium-like resonances have been found, **4q needed**

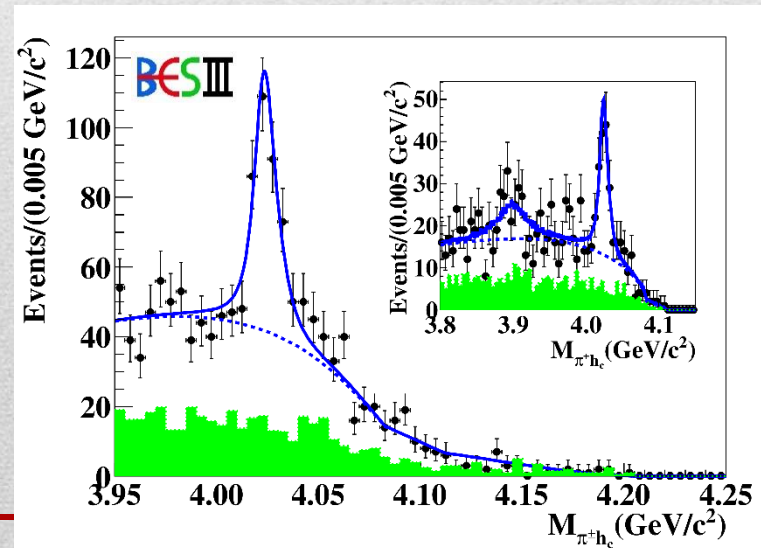
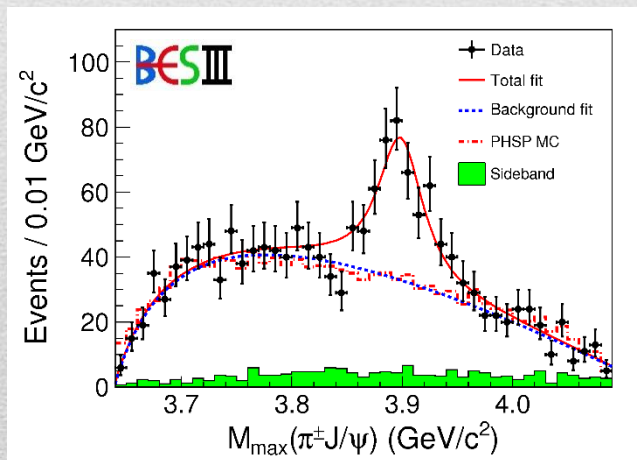
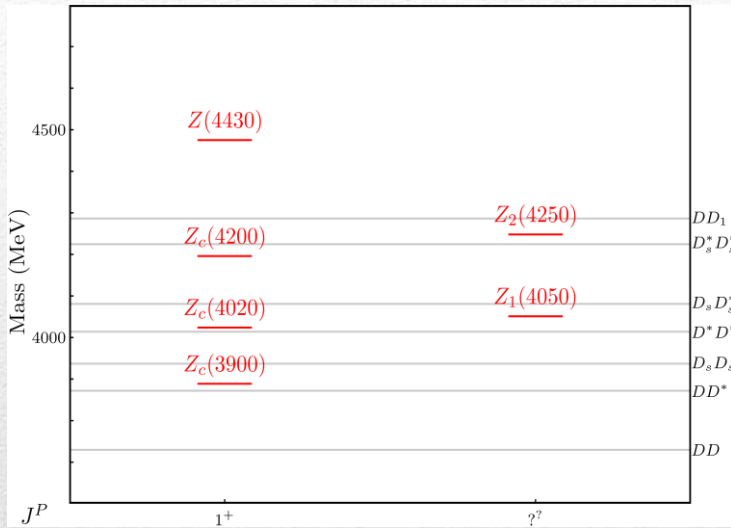
Two states $J^{PC} = 1^{+-}$ appear
slightly above $D^{(*)}D^*$ thresholds

$$e^+e^- \rightarrow Z_c(3900)^+\pi^- \rightarrow J/\psi \pi^+\pi^- \text{ and } (DD^*)^+\pi^-$$

$$M = 3888.7 \pm 3.4 \text{ MeV}, \Gamma = 35 \pm 7 \text{ MeV}$$

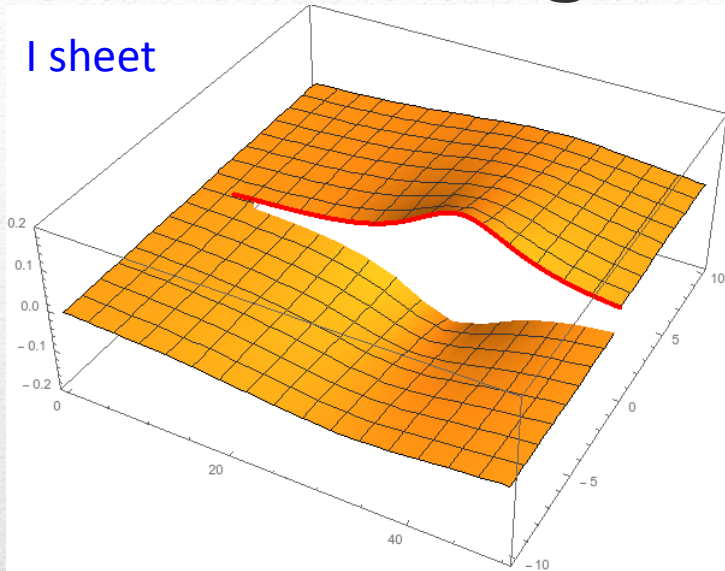
$$e^+e^- \rightarrow Z_c'(4020)^+\pi^- \rightarrow h_c \pi^+\pi^- \text{ and } \bar{D}^{*0}D^{*+}\pi^-$$

$$M = 4023.9 \pm 2.4 \text{ MeV}, \Gamma = 10 \pm 6 \text{ MeV}$$

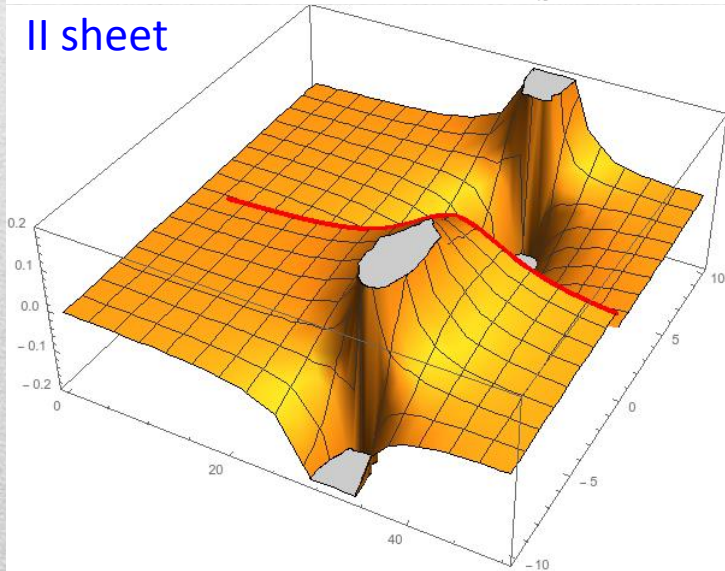


Pole hunting

I sheet

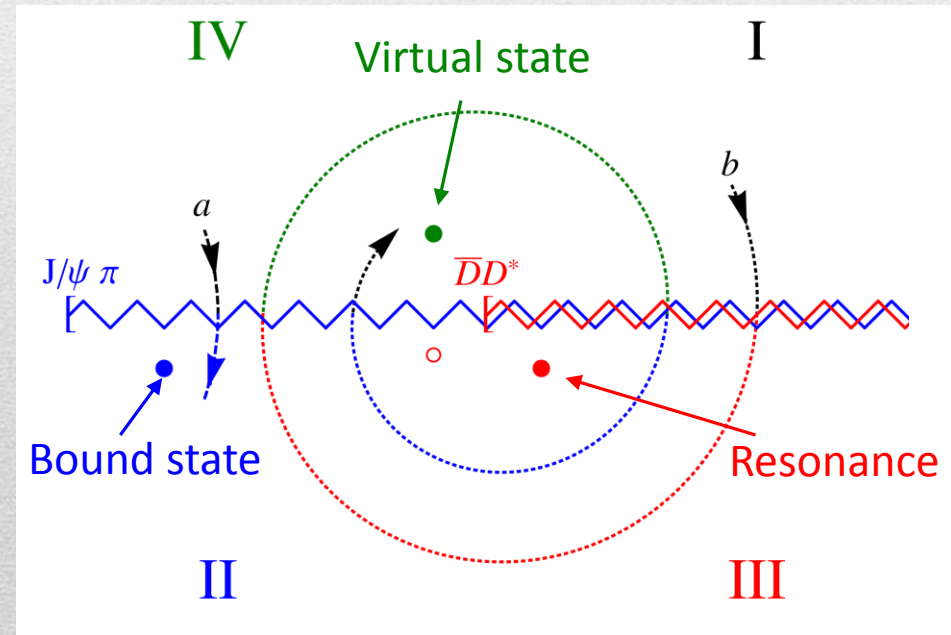


II sheet

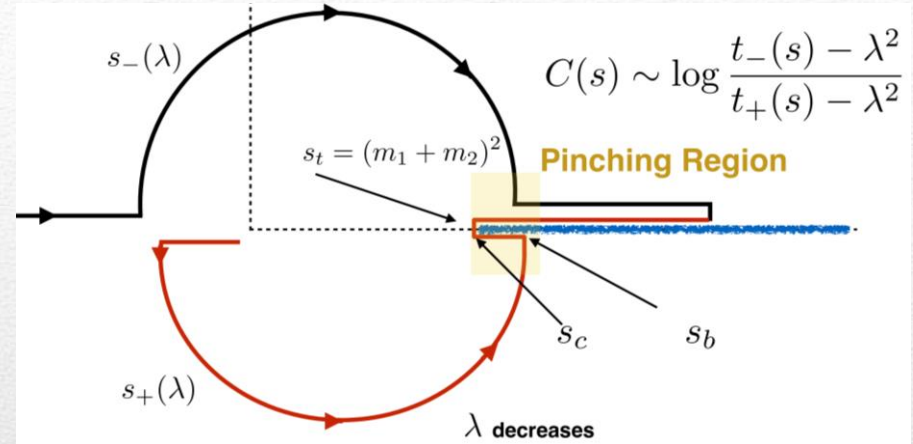
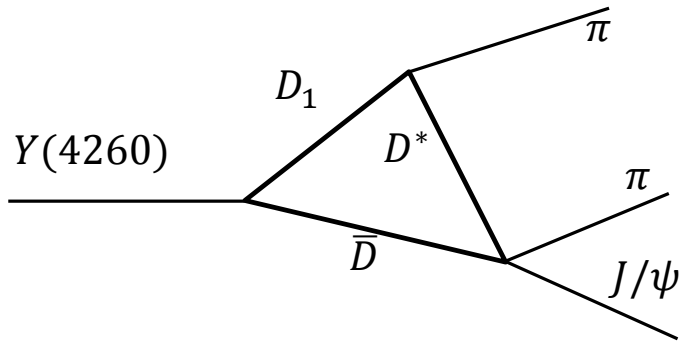


More complicated structure when more thresholds arise:
two sheets for each new threshold

III sheet: usual resonances
IV sheet: cusps (virtual states)



Triangle singularity



Logarithmic branch points due to exchanges in the cross channels can simulate a resonant behavior, only in **very special kinematical conditions** (Coleman and Norton, *Nuovo Cim.* 38, 438), However, this effects **cancel in Dalitz projections, no peaks** (Schmid, *Phys.Rev.* 154, 1363)

$$f_{0,i}(s) = b_{0,i}(s) + \frac{t_{ij}}{\pi} \int_{s_i}^{\infty} ds' \frac{\rho_j(s') b_{0,j}(s')}{s' - s}$$

...but the cancellation can be spread in different channels, you might still see peaks in other channels only!

Szczepaniak, PLB747, 410-416

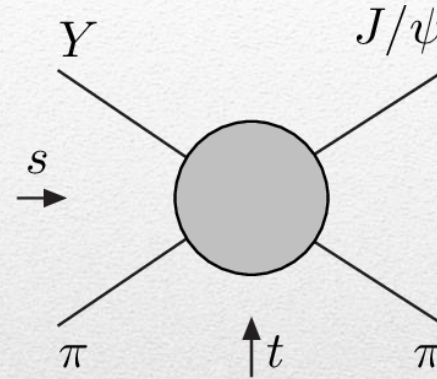
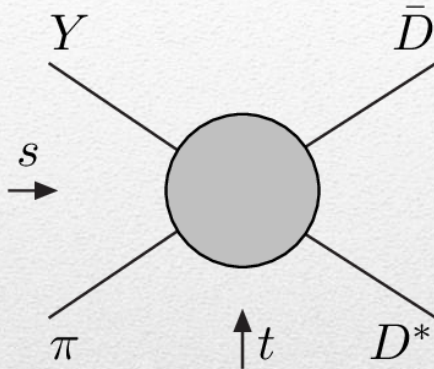
Szczepaniak, PLB757, 61-64

Guo, Meissner, Wang, Yang PRD92, 071502

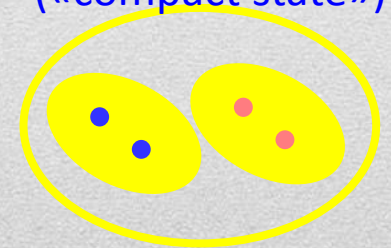
Amplitude analysis for $Z_c(3900)$

One can test different parametrizations of the amplitude, which correspond to different singularities \rightarrow different natures

AP et al. (JPAC), PLB772, 200-209



Resonance,
III sheet pole
 («compact state»)



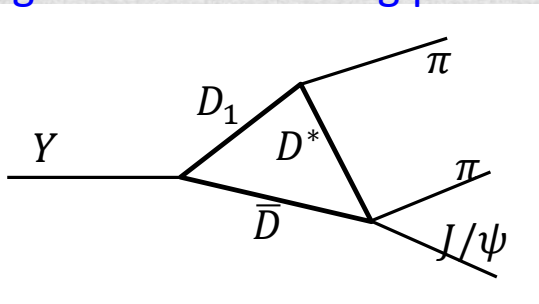
Maiani et al., PRD71, 014028
Faccini et al., PRD87, 111102
Esposito et al., Phys.Rept. 668

(anti)bound state,
II/IV sheet pole
 («molecule»)



Tornqvist, Z.Phys. C61, 525
Swanson, Phys.Rept. 429
Hanhart et al. PRL111, 132003

Triangle rescattering,
logarithmic branching point



Szczepaniak, PLB747, 410-416
Szczepaniak, PLB757, 61-64
Guo et al. PRD92, 071502

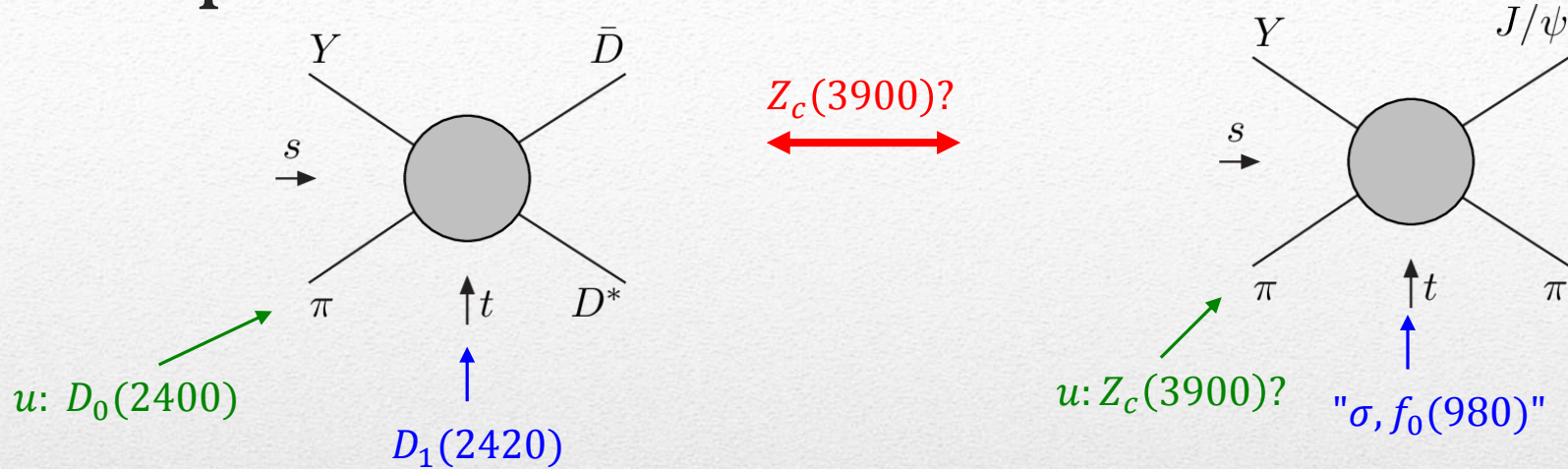
Strategy

- We fit the following **invariant mass distributions**:
 - BESIII PRL110, 252001 $J/\psi \pi^+, J/\psi \pi^-, \pi^+ \pi^-$ at $E_{CM} = 4.26$ GeV
 - BESIII PRL110, 252001 $J/\psi \pi^0$ at $E_{CM} = 4.23, 4.26, 4.36$ GeV
 - BESIII PRD92, 092006 $\overline{D^0} D^{*+}, \overline{D^{*0}} D^+$ (double tag) at $E_{CM} = 4.23, 4.26$ GeV
 - BESIII PRL115, 222002 $\overline{D^0} D^{*0}, \overline{D^{*0}} D^0$ at $E_{CM} = 4.23, 4.26$ GeV
 - ~~BESIII PRL112, 022001 $\overline{D^0} D^{*+}, \overline{D^{*0}} D^+$ (single tag) at $E_{CM} = 4.26$ GeV~~
 - ~~Belle PRL110, 252002 $J/\psi \pi^\pm$ at $E_{CM} = 4.26$ GeV~~
 - ~~CLEO-c data PLB727, 366 $J/\psi \pi^\pm, J/\psi \pi^0$ at $E_{CM} = 4.17$ GeV~~
- Published data are not efficiency/acceptance corrected,
→ we are **not able to give the absolute normalization** of the amplitudes
- No given dependence on E_{CM} is assumed – the couplings at different E_{CM} are independent parameters

Strategy

- **Reducible** (incoherent) **backgrounds are pretty flat** and do not influence the analysis, except the peaking background in $\overline{D^0 D^{*0}}, \overline{D^{*0} D^0}$ (subtracted)
- Some information about **angular distributions** has been published, but it's **not constraining** enough → we do not include in the fit
- Because of that, **we approximate all the particles to be scalar** – this affects the value of couplings, which are not normalized anyway – but not the position of singularities. This also limits the number of free parameters

Amplitude model



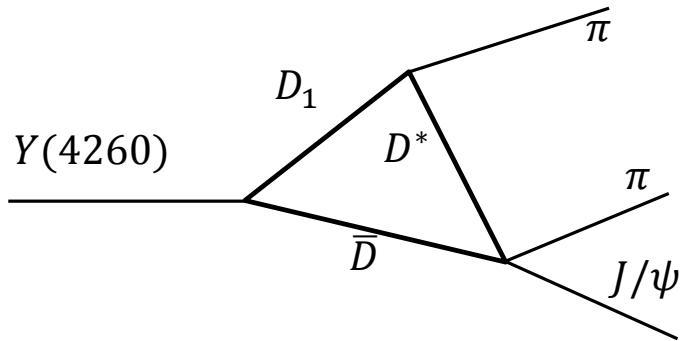
$$f_i(s, t, u) = 16\pi \sum_{l=0}^{L_{\max}} (2l+1) \left(a_{l,i}^{(s)}(s) P_l(z_s) + a_{l,i}^{(t)}(t) P_l(z_t) + a_{l,i}^{(u)}(u) P_l(z_u) \right) \quad \text{Khuri-Treiman}$$

$$f_{0,i}(s) = \frac{1}{32\pi} \int_{-1}^1 dz_s f_i(s, t(s, z_s), u(s, z_s)) = a_{0,i}^{(s)} + \frac{1}{32\pi} \int_{-1}^1 dz_s \left(a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) \right) \equiv a_{0,i}^{(s)} + b_{0,i}(s)$$

$$f_{l,i}(s) = \frac{1}{32\pi} \int_{-1}^1 dz_s P_l(z_s) \left(a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) \right) \equiv b_{l,i}(s) \quad \text{for } l > 0. \quad f_{0,i}(s) = b_{0,i}(s) + \sum_j t_{ij}(s) \frac{1}{\pi} \int_{s_j}^{\infty} ds' \frac{\rho_j(s') b_{0,j}(s')}{s' - s},$$

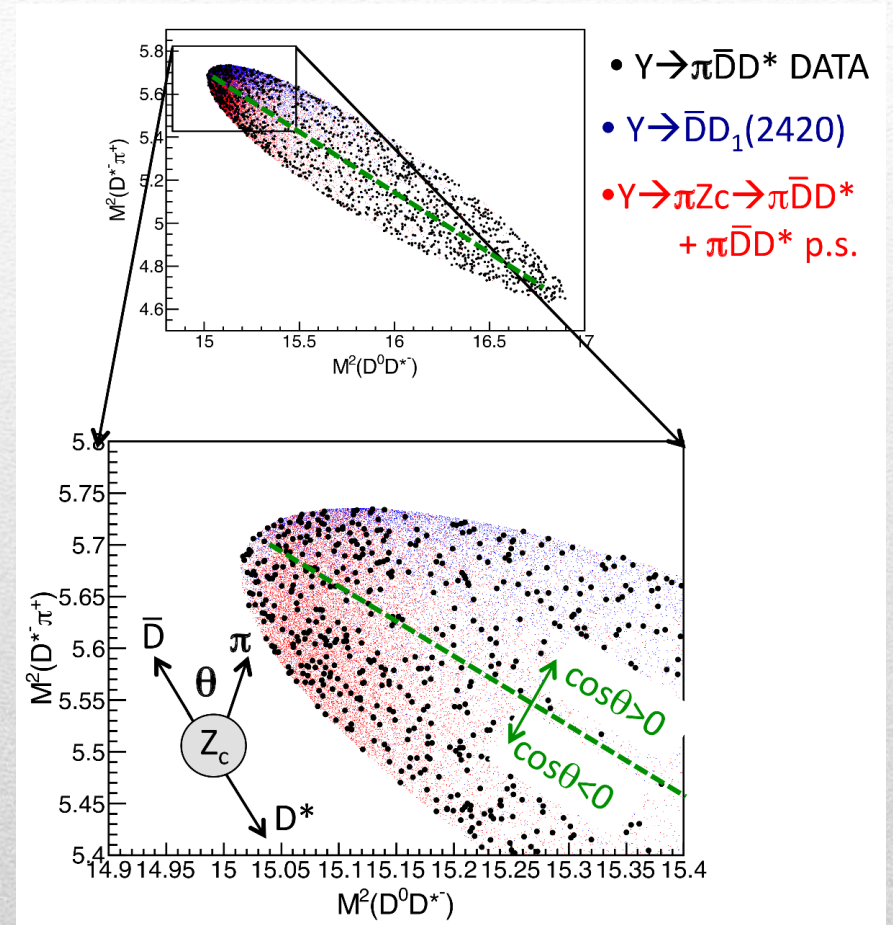
$$f_i(s, t, u) = 16\pi \left[a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) + \sum_j t_{ij}(s) \left(c_j + \frac{s}{\pi} \int_{s_j}^{\infty} ds' \frac{\rho_j(s') b_{0,j}(s')}{s' (s' - s)} \right) \right],$$

Triangle singularity



The dominance of $\bar{D}D_1$ in the $Y(4260)$ decay is **neither supported nor disproved** by data – the measurement of the asymmetry of the angular distribution across the Dalitz plot is inconclusive

Higher statistics will allow to constrain the $Y\bar{D}D_1$ coupling, and consequently the intensity of the triangle singularity



Testing scenarios

- We approximate all the particles to be scalar – this affects the value of couplings, which are not normalized anyway – but not the position of singularities. This also limits the number of free parameters

$$f_i(s, t, u) = 16\pi \left[a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) + \sum_j t_{ij}(s) \left(c_j + \frac{s}{\pi} \int_{s_j}^{\infty} ds' \frac{\rho_j(s') b_{0,j}(s')}{s' (s' - s)} \right) \right],$$

The scattering matrix is parametrized as $(t^{-1})_{ij} = K_{ij} - i \rho_i \delta_{ij}$

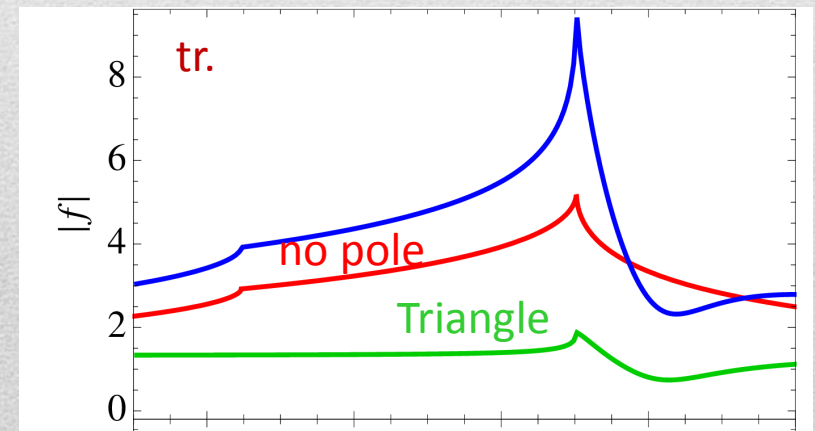
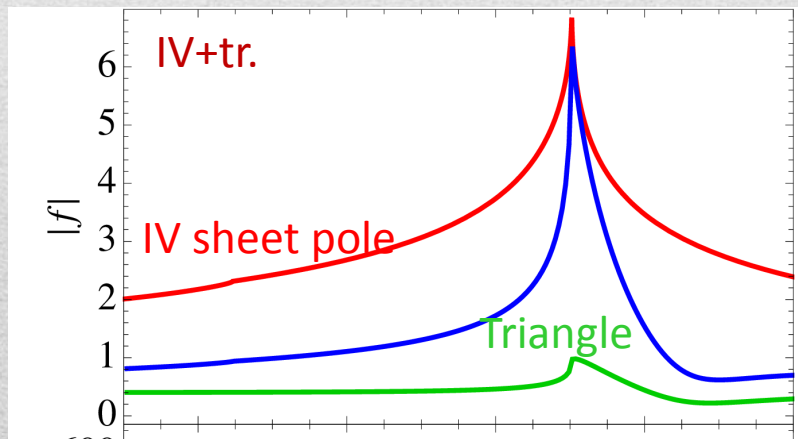
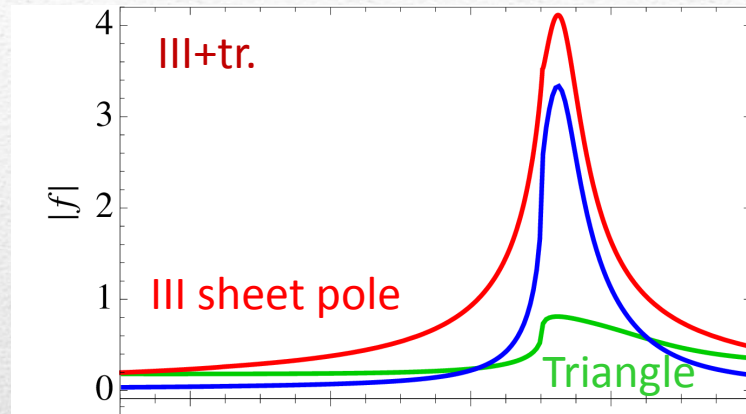
Four different scenarios considered:

- «III»: the K matrix is $\frac{g_i g_j}{M^2 - s}$, this generates a pole in the closest unphysical sheet the rescattering integral is set to zero
- «III+tr.»: same, but with the correct value of the rescattering integral
- «IV+tr.»: the K matrix is constant, this generates a pole in the IV sheet
- «tr.»: same, but the pole is pushed far away by adding a penalty in the χ^2

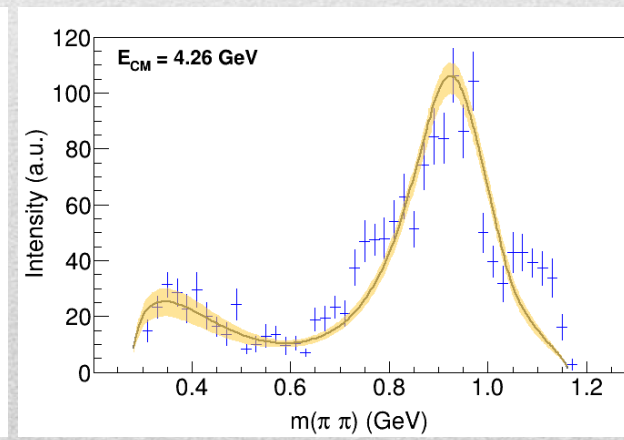
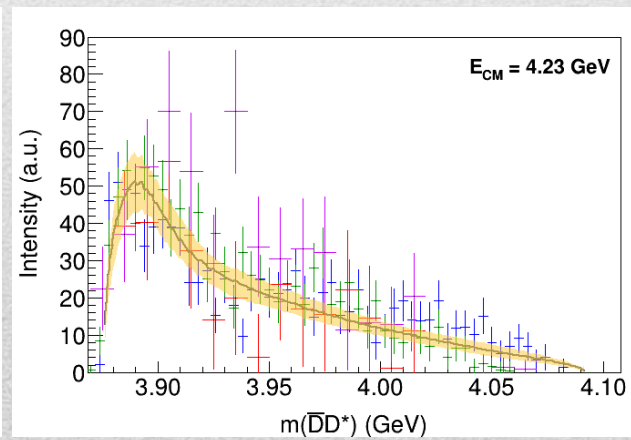
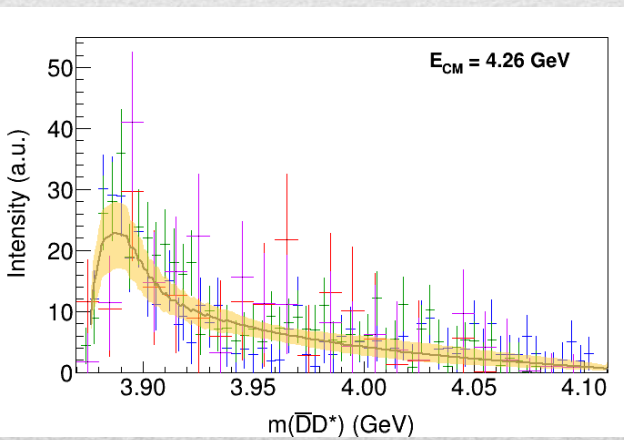
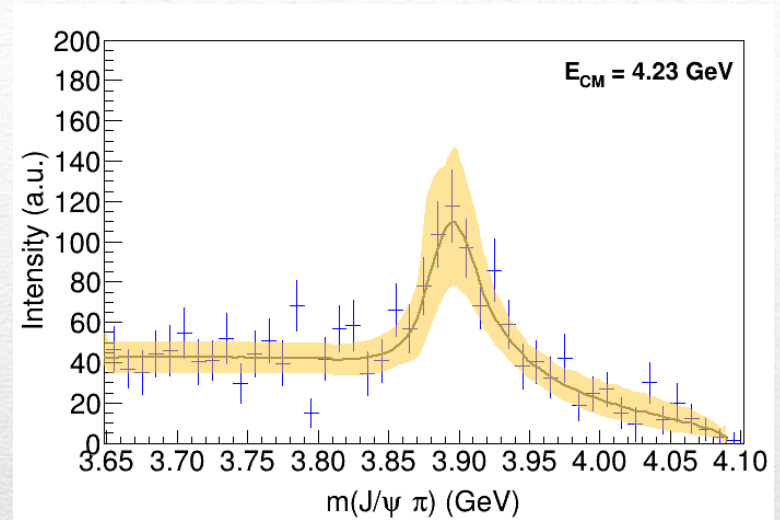
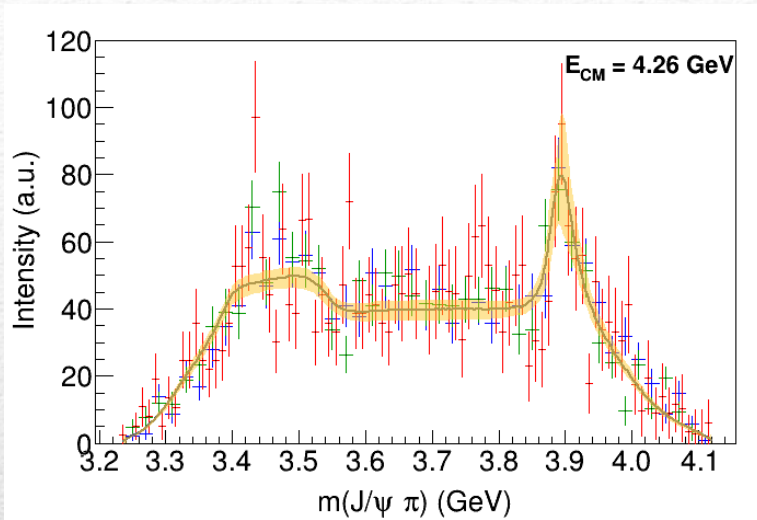
Singularities and lineshapes

Different lineshapes according to different singularities

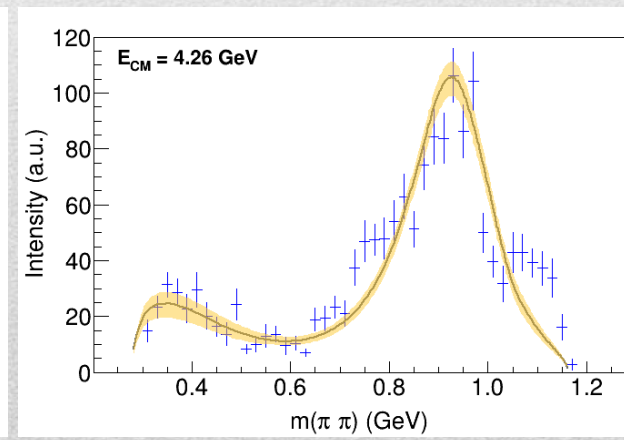
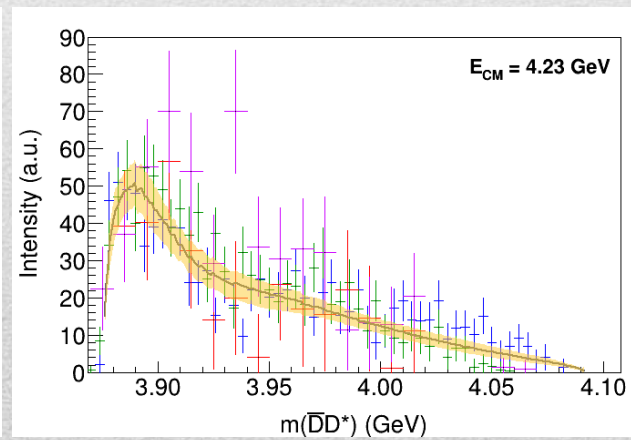
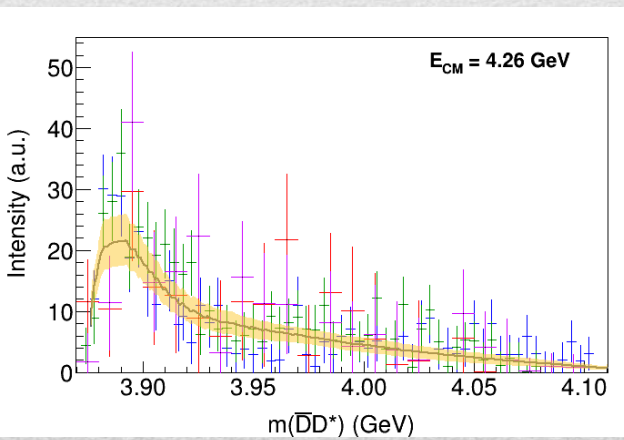
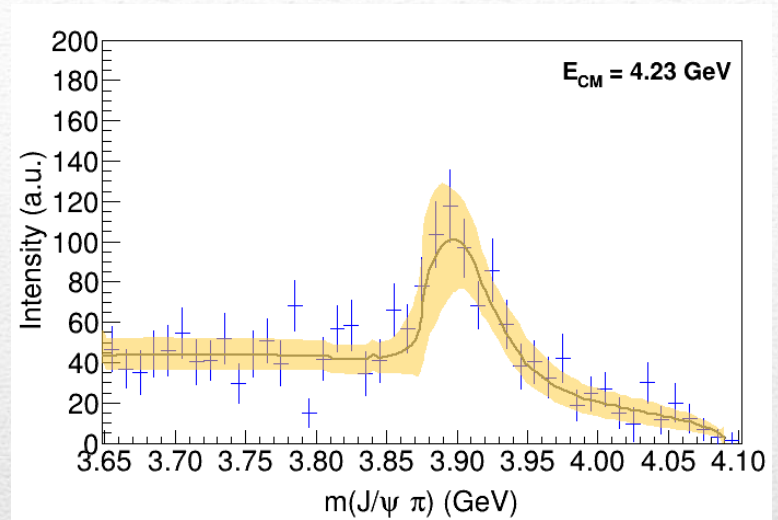
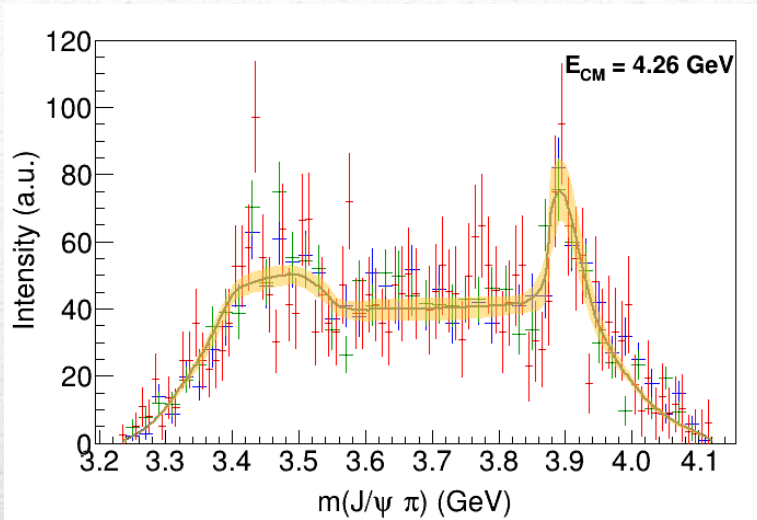
— Triangle
— t matrix
— Full



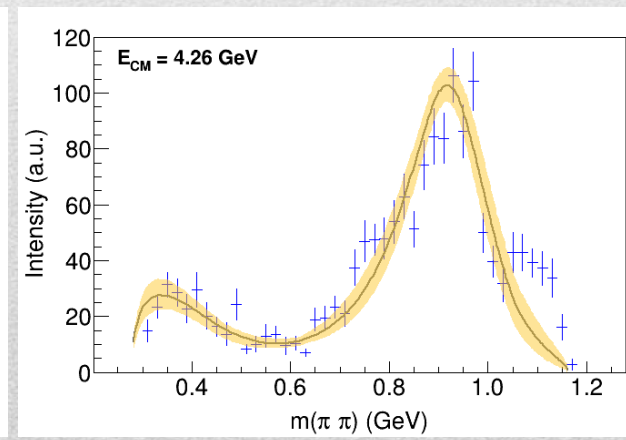
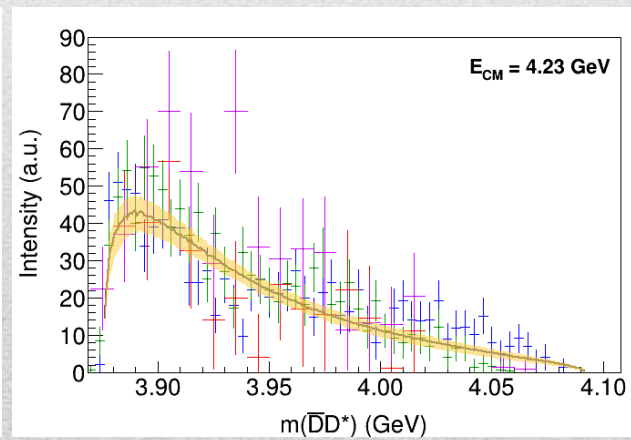
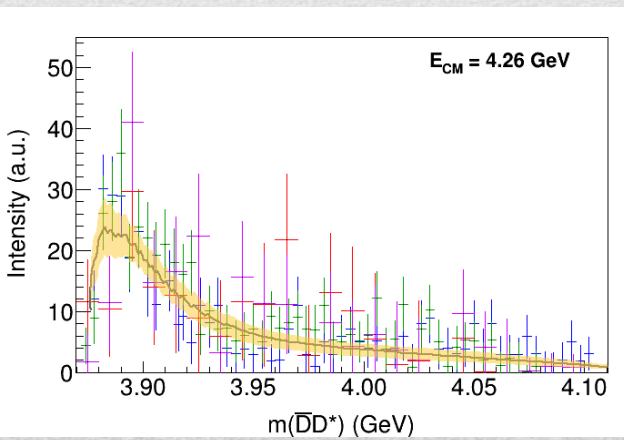
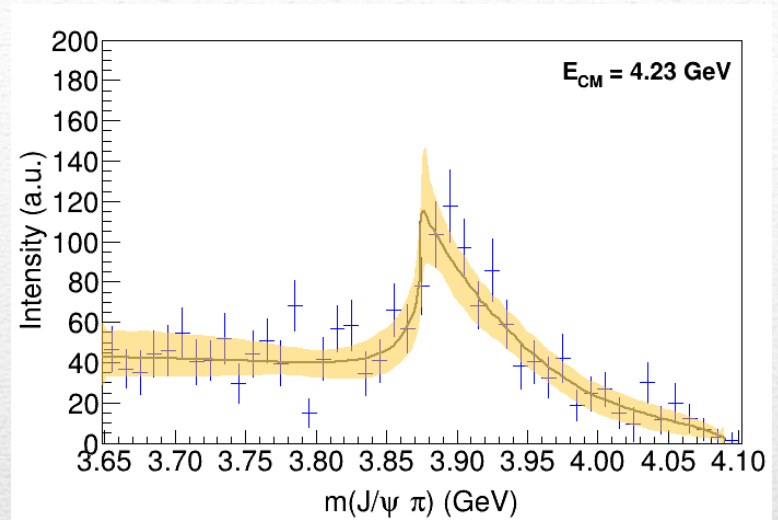
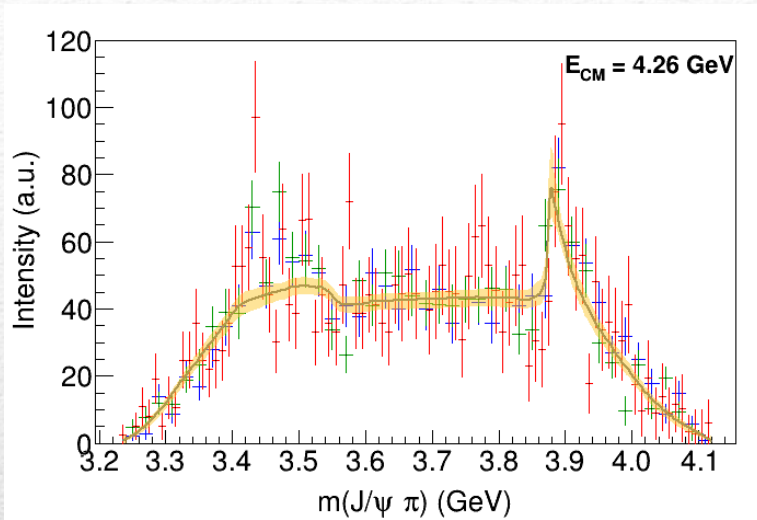
Fit: III



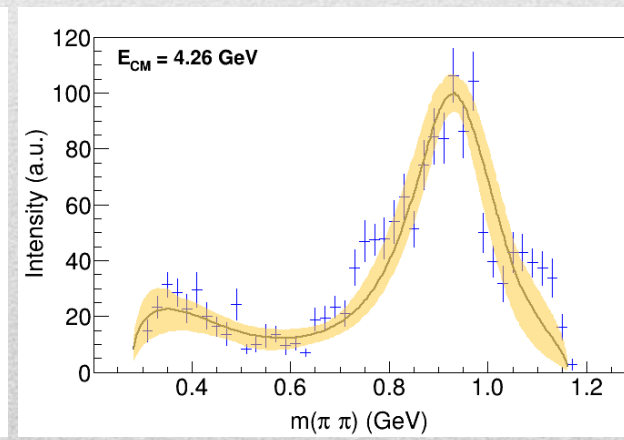
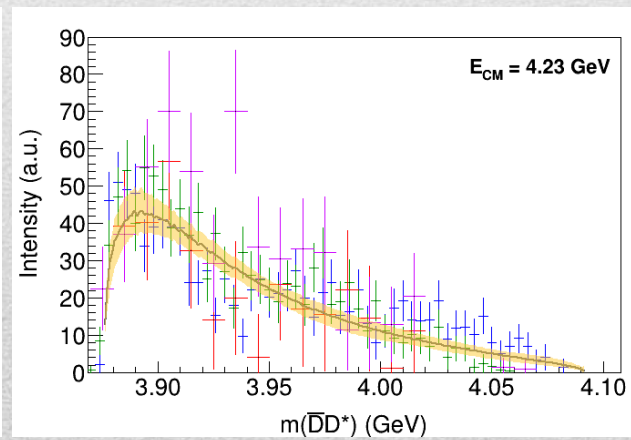
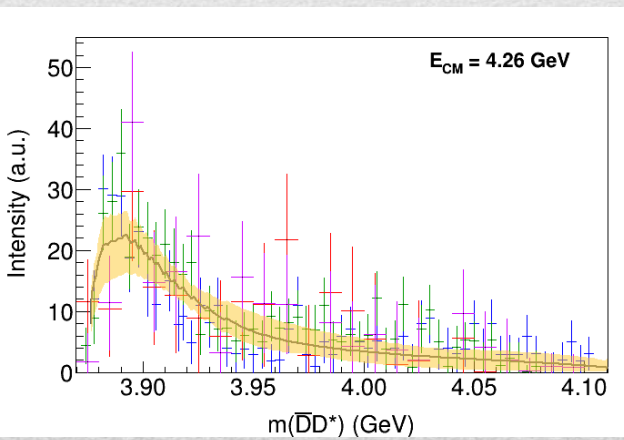
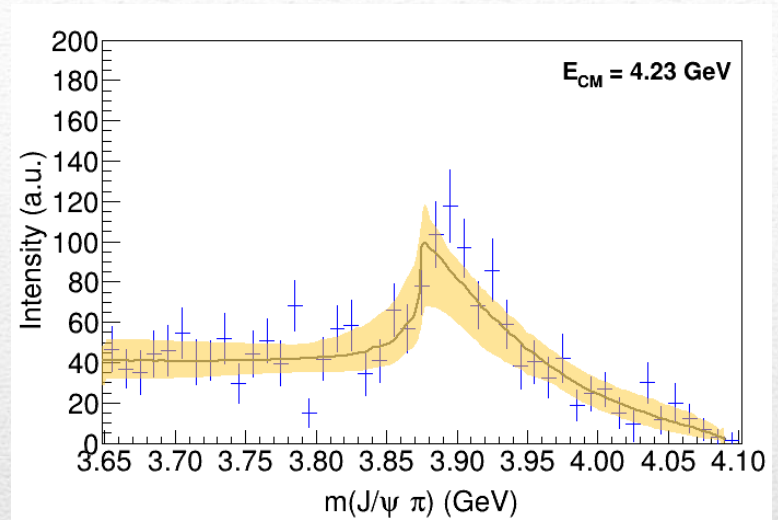
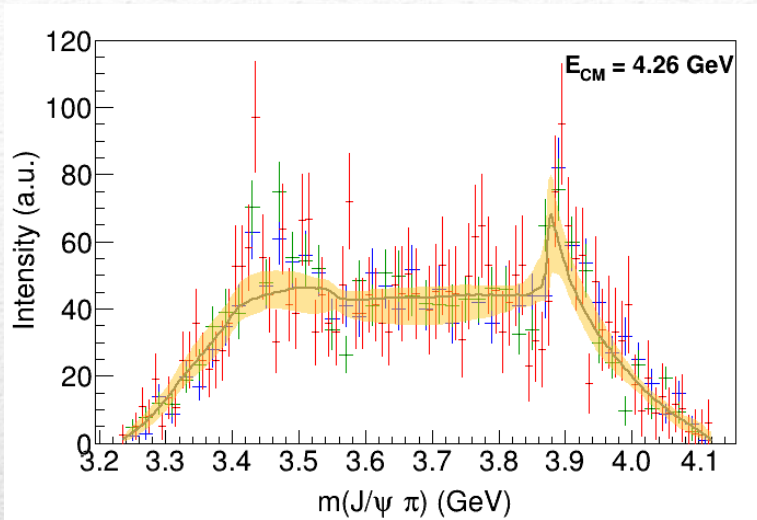
Fit: III+tr.



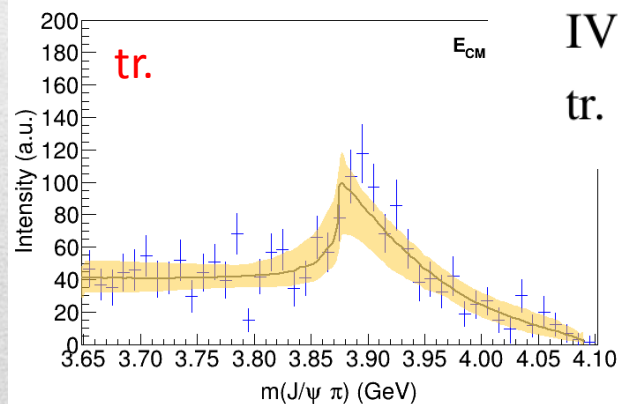
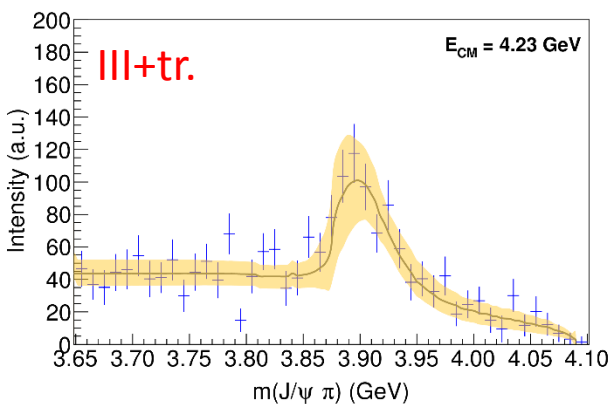
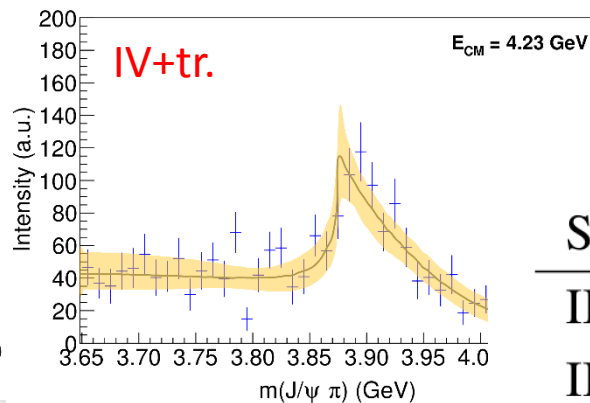
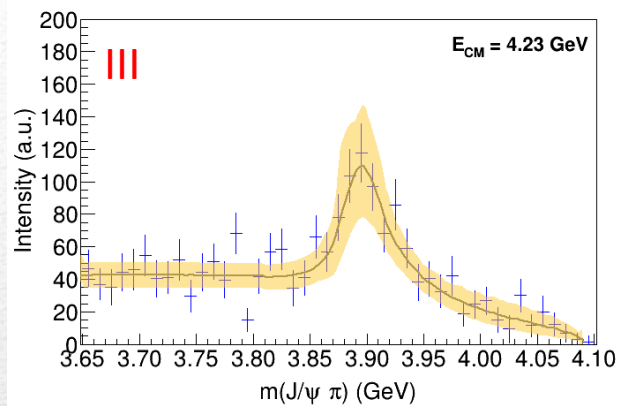
Fit: IV+tr.



Fit: tr.



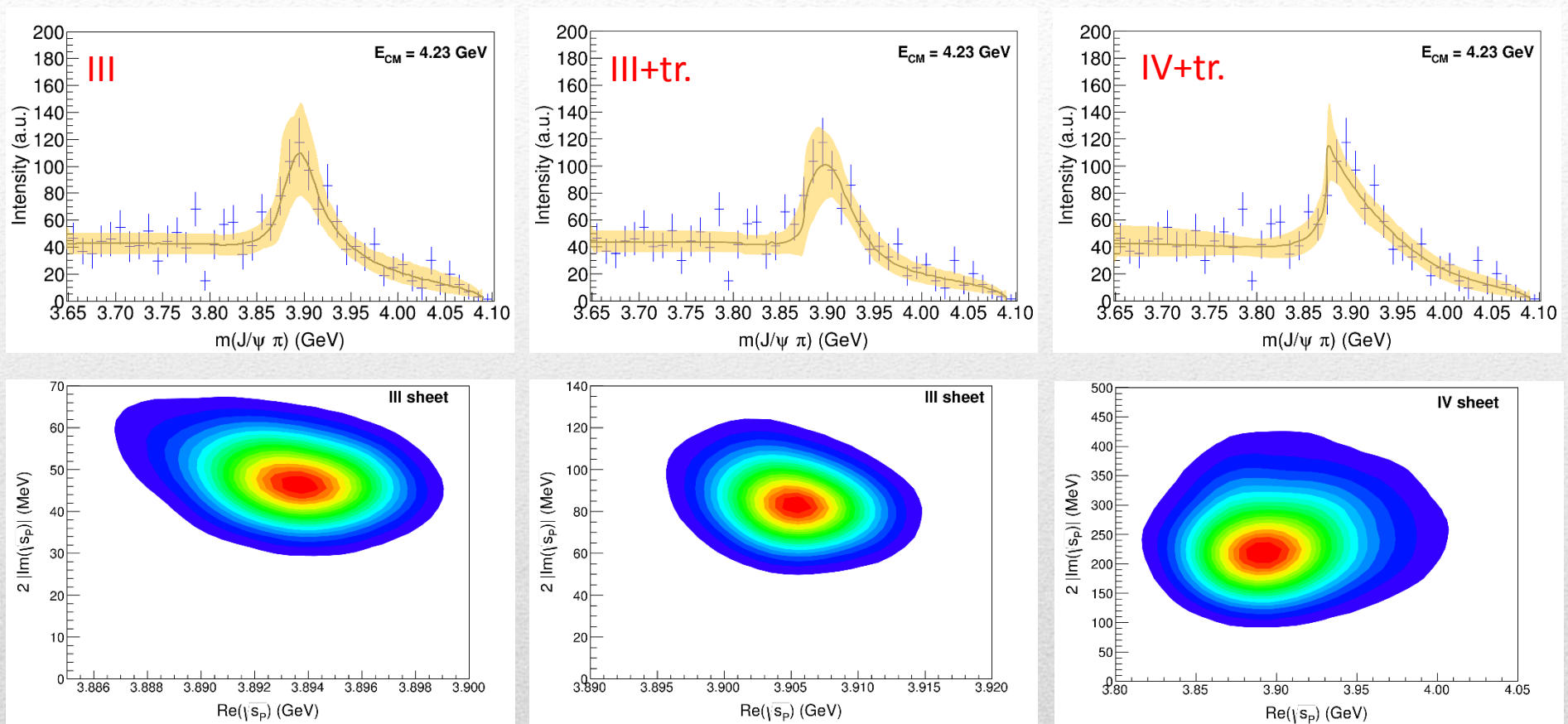
Fit summary



Scenario	χ^2	DOF	χ^2/DOF
III	644	532	1.21
III+tr.	642	532	1.21
IV+tr.	666	532	1.25
tr.	695	532	1.31

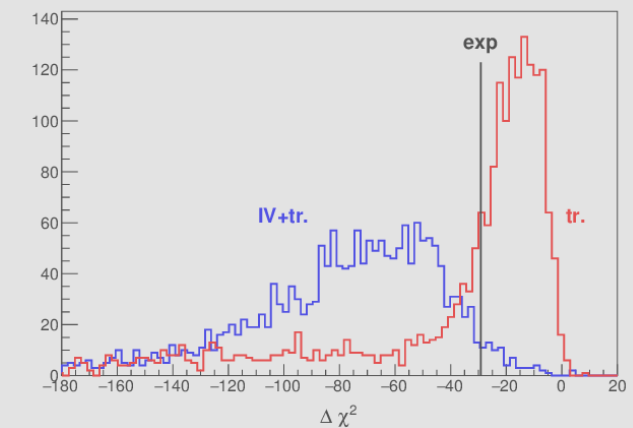
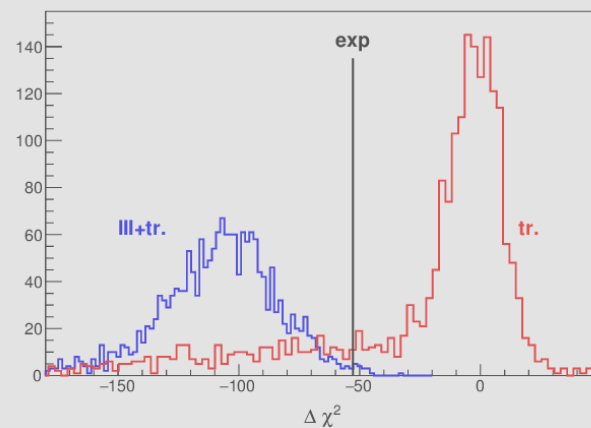
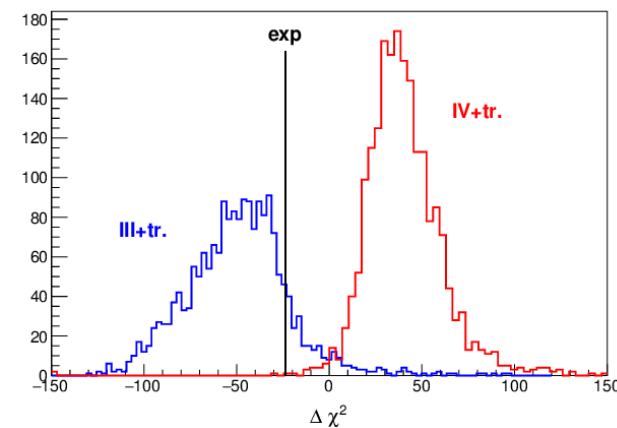
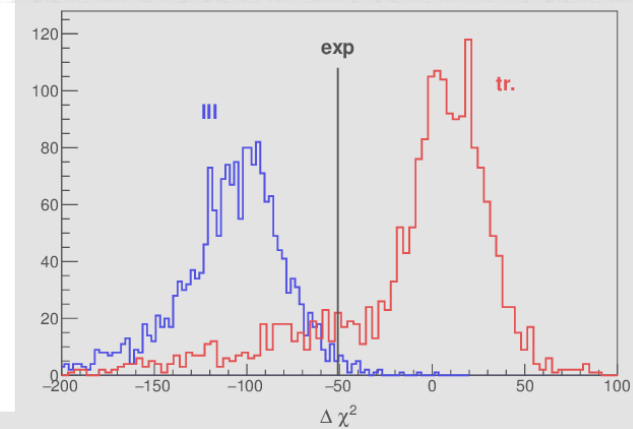
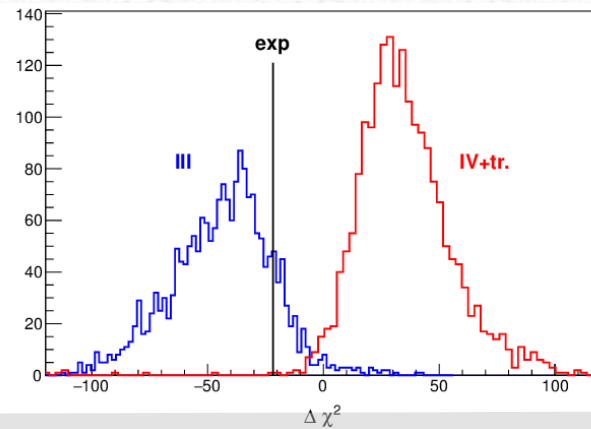
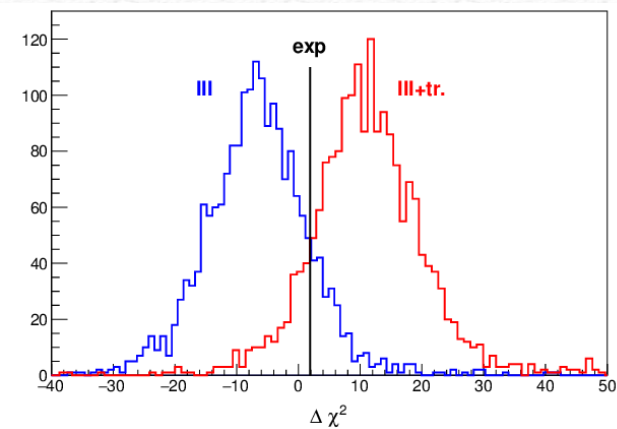
Naive loglikelihood ratio test give a $\sim 4\sigma$ significance of the scenario III+tr. over IV+tr., looking at plots it looks too much – better using some more solid test

Pole extraction



	III	III+tr.	IV+tr.
M (MeV)	$3893.2^{+5.5}_{-7.7}$	3905^{+11}_{-9}	3900^{+140}_{-90}
Γ (MeV)	48^{+19}_{-14}	85^{+45}_{-26}	240^{+230}_{-130}

Statistical analysis

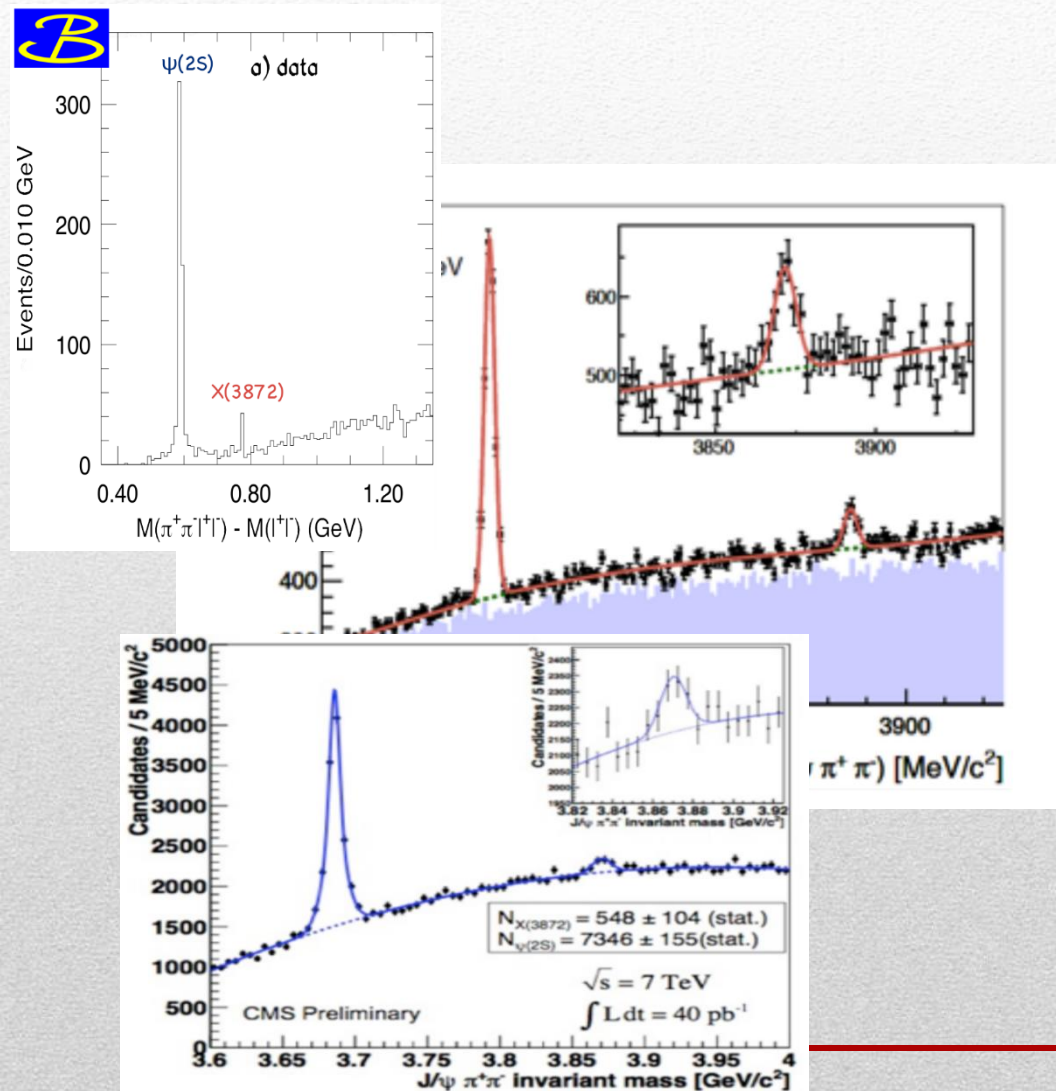


Toy experiments according to the different hypotheses, to estimate the relative rejection of various scenarios

Scenario	III+tr.	IV+tr.	tr.
III	1.5σ (1.5σ)	1.5σ (2.7σ)	" 2.4σ " (" 1.4σ ")
III+tr.	—	1.5σ (3.1σ)	" 2.6σ " (" 1.3σ ")
IV+tr.			" 2.1σ " (" 0.9σ ")

Not conclusive at this stage

X(3872)



- Discovered in $B \rightarrow K X \rightarrow K J/\psi \pi \pi$
- Quantum numbers 1^{++}
- **Very close** to DD^* threshold
- **Too narrow** for an above-threshold charmonium
- **Isospin violation** too big $\frac{\Gamma(X \rightarrow J/\psi \omega)}{\Gamma(X \rightarrow J/\psi \rho)} \sim 0.8 \pm 0.3$
- **Mass** prediction not compatible with $\chi_{c1}(2P)$

$$M = 3871.68 \pm 0.17 \text{ MeV}$$

$$M_X - M_{DD^*} = -3 \pm 192 \text{ keV}$$

$$\Gamma < 1.2 \text{ MeV @90\%}$$

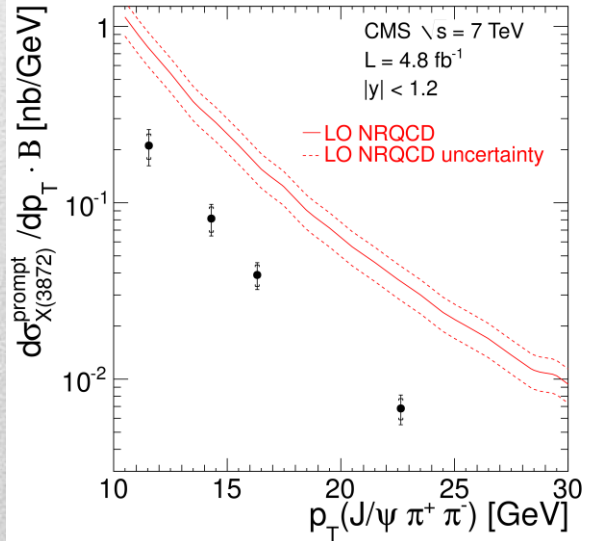
X(3872)

Large prompt production
at hadron colliders

$$\sigma_B/\sigma_{TOT} = (26.3 \pm 2.3 \pm 1.6)\%$$

$$\sigma_{PR} \times B(X \rightarrow J/\psi\pi\pi) = (1.06 \pm 0.11 \pm 0.15) \text{ nb}$$

CMS, JHEP 1304, 154



B decay mode	X decay mode	product branching fraction ($\times 10^5$)		B_{fit}	R_{fit}
$K^+ X$	$X \rightarrow \pi\pi J/\psi$	0.86 ± 0.08	(BABAR ^[26] Belle ^[25])	$0.081^{+0.019}_{-0.031}$	1
		$0.84 \pm 0.15 \pm 0.07$	BABAR ^[26]		
		$0.86 \pm 0.08 \pm 0.05$	Belle ^[25]		
$K^0 X$	$X \rightarrow \pi\pi J/\psi$	0.41 ± 0.11	(BABAR ^[26] Belle ^[25])		
		$0.35 \pm 0.19 \pm 0.04$	BABAR ^[26]		
		$0.43 \pm 0.12 \pm 0.04$	Belle ^[25]		
$(K^+\pi^-)_{NR} X$	$X \rightarrow \pi\pi J/\psi$	$0.81 \pm 0.20^{+0.11}_{-0.14}$	Belle ^[106]		
$K^{*0} X$	$X \rightarrow \pi\pi J/\psi$	< 0.34 , 90% C.L.	Belle ^[106]		
KX	$X \rightarrow \omega J/\psi$	$R = 0.8 \pm 0.3$	BABAR ^[33]	$0.061^{+0.024}_{-0.036}$	$0.77^{+0.28}_{-0.32}$
$K^+ X$		$0.6 \pm 0.2 \pm 0.1$	BABAR ^[33]		
$K^0 X$		$0.6 \pm 0.3 \pm 0.1$	BABAR ^[33]		
KX	$X \rightarrow \pi\pi\pi^0 J/\psi$	$R = 1.0 \pm 0.4 \pm 0.3$	Belle ^[32]		
$K^+ X$	$X \rightarrow D^{*0} \bar{D}^0$	8.5 ± 2.6	(BABAR ^[38] Belle ^[37])	$0.614^{+0.166}_{-0.074}$	$8.2^{+2.3}_{-2.8}$
		$16.7 \pm 3.6 \pm 4.7$	BABAR ^[38]		
		$7.7 \pm 1.6 \pm 1.0$	Belle ^[37]		
$K^0 X$	$X \rightarrow D^{*0} \bar{D}^0$	12 ± 4	(BABAR ^[38] Belle ^[37])		
		$22 \pm 10 \pm 4$	BABAR ^[38]		
		$9.7 \pm 4.6 \pm 1.3$	Belle ^[37]		
$K^+ X$	$X \rightarrow \gamma J/\psi$	0.202 ± 0.038	(BABAR ^[35] Belle ^[34])	$0.019^{+0.005}_{-0.009}$	$0.24^{+0.05}_{-0.06}$
$K^+ X$		$0.28 \pm 0.08 \pm 0.01$	BABAR ^[35]		
$K^0 X$		$0.178^{+0.048}_{-0.044} \pm 0.012$	Belle ^[34]		
		$0.26 \pm 0.18 \pm 0.02$	BABAR ^[35]		
$K^0 X$		$0.124^{+0.076}_{-0.061} \pm 0.011$	Belle ^[34]		
$K^+ X$	$X \rightarrow \gamma\psi(2S)$	0.44 ± 0.12	BABAR ^[35]	$0.04^{+0.015}_{-0.020}$	$0.51^{+0.13}_{-0.17}$
		$0.95 \pm 0.27 \pm 0.06$	BABAR ^[35]		
		$0.083^{+0.198}_{-0.183} \pm 0.044$	Belle ^[34]		
$K^0 X$		$R' = 2.46 \pm 0.64 \pm 0.29$	LHCb ^[36]		
		$1.14 \pm 0.55 \pm 0.10$	BABAR ^[35]		
		$0.112^{+0.357}_{-0.290} \pm 0.057$	Belle ^[34]		
$K^+ X$	$X \rightarrow \gamma\chi_{c1}$	$< 9.6 \times 10^{-3}$	Belle ^[23]	$< 1.0 \times 10^{-3}$	< 0.014
$K^+ X$	$X \rightarrow \gamma\chi_{c2}$	< 0.016	Belle ^[23]	$< 1.7 \times 10^{-3}$	< 0.024
KX	$X \rightarrow \gamma\gamma$	$< 4.5 \times 10^{-3}$	Belle ^[111]	$< 4.7 \times 10^{-4}$	$< 6.6 \times 10^{-3}$
KX	$X \rightarrow \eta J/\psi$	< 1.05	BABAR ^[112]	< 0.11	< 1.55
$K^+ X$	$X \rightarrow p\bar{p}$	$< 9.6 \times 10^{-4}$	LHCb ^[110]	$< 1.6 \times 10^{-4}$	$< 2.2 \times 10^{-3}$

Weinberg theorem

Resonant scattering amplitude

$$f(ab \rightarrow c \rightarrow ab) = -\frac{1}{8\pi E_{CM}} g^2 \frac{1}{(p_a + p_b)^2 - m_c^2}$$

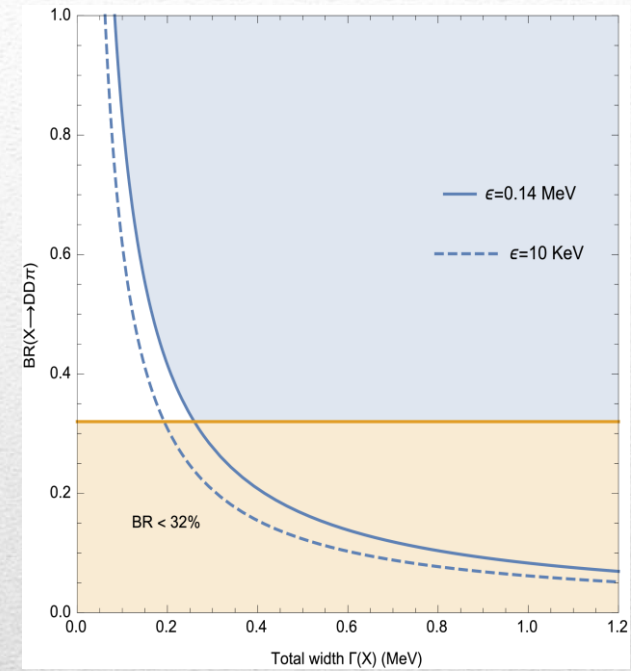
with $m_c = m_a + m_b - B$, and $B, T \ll m_{a,b}$

$$f(ab \rightarrow c \rightarrow ab) = -\frac{1}{16\pi(m_a + m_b)^2} g^2 \frac{1}{B + T}$$

This has to be compared with the potential scattering for slow particles ($kR \ll 1$, being $R \sim 1/m_\pi$ the range of interaction) in an attractive potential U with a superficial level at $-B$

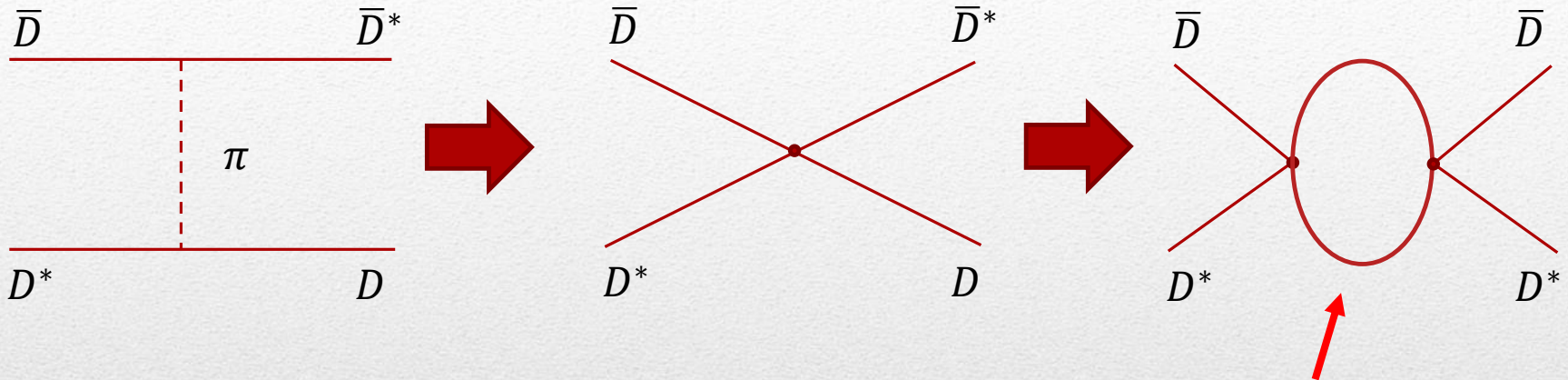
$$f(ab \rightarrow ab) = -\frac{1}{\sqrt{2\mu}} \frac{\sqrt{B} - i\sqrt{T}}{B + T}, \quad B = \frac{g^4}{512\pi^2} \frac{\mu^5}{(m_a m_b)^2}$$

This corresponds to the pure molecular interpretation of the $X(3872)$



Weinberg, PR 130, 776
Weinberg, PR 137, B672
Polosa, PLB 746, 248

Weinberg and amplitudes



This means that IF you can consider the pion exchange as a contact interaction, the amplitude is determined by the pole close to threshold

This loop is now divergent, I need to renormalize the integral I can put the pole where I want

Complex s



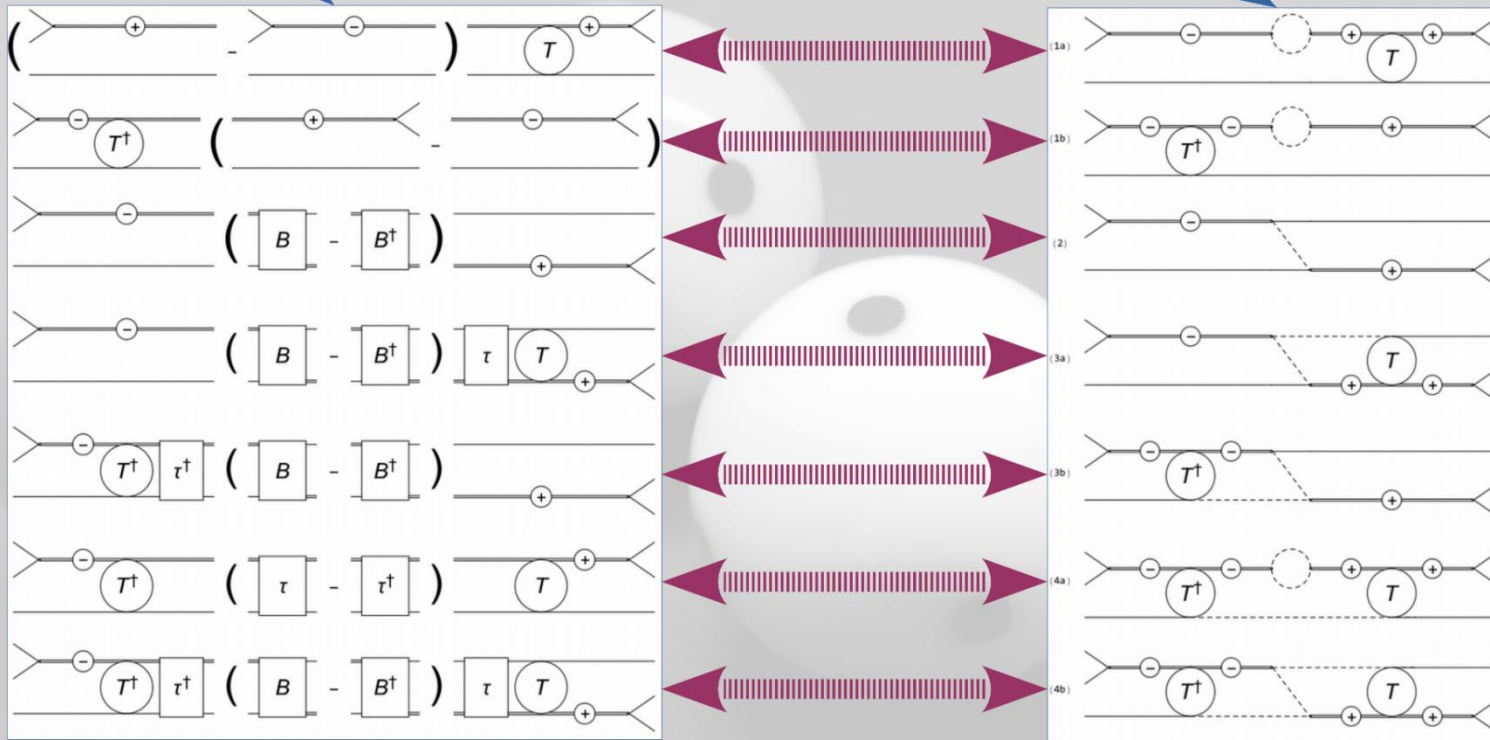
Three-Body Unitarity

Mai, Hu, Doring, AP, Szczepaniak, EPJA53, 9, 177

See talk by M. Doring tomorrow

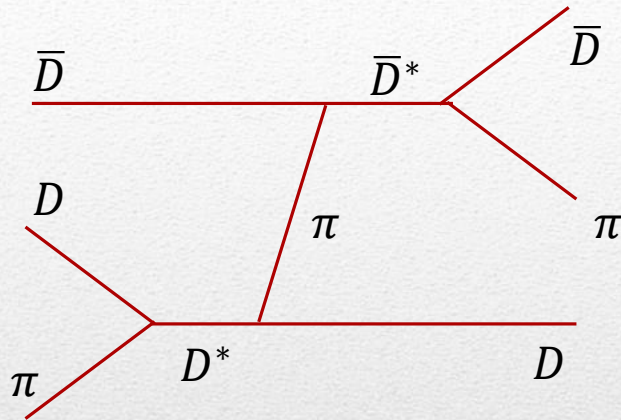
3-body Unitarity (normalization condition \leftrightarrow phase space integral)

$$\langle q_1, q_2, q_3 | (\hat{T} - \hat{T}^\dagger) | p_1, p_2, p_3 \rangle = i \int_P \langle q_1, q_2, q_3 | \hat{T}^\dagger | k_1, k_2, k_3 \rangle \langle k_1, k_2, k_3 | \hat{T} | p_1, p_2, p_3 \rangle$$



Weinberg and amplitudes

A. Jackura et al., in progress



BUT the D^* actually decays into $D\pi$ and the system is constrained by 3-body unitarity

The position of the pole can be calculated given a model for the simple pion exchange

The simplest model leads to a convergent dispersion relation, the pole position is determined
One can check whether this purely molecular amplitude is consistent or not with data

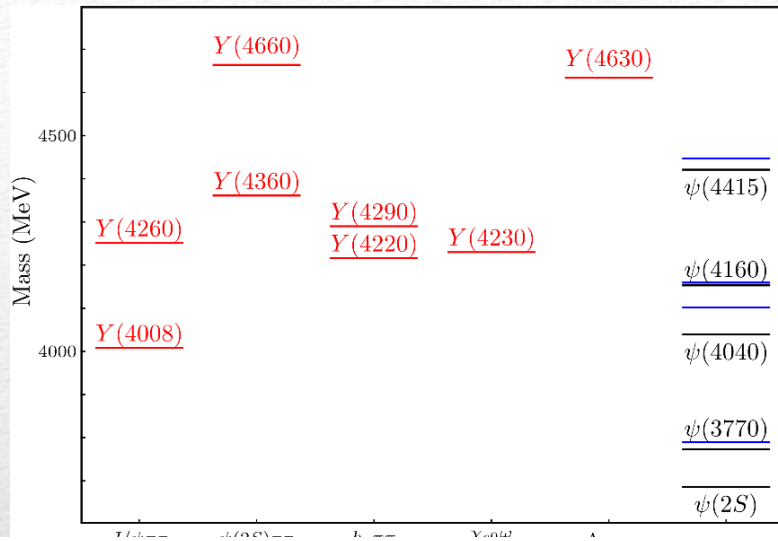
Complex s

Short cut of real pion exchange



pole?

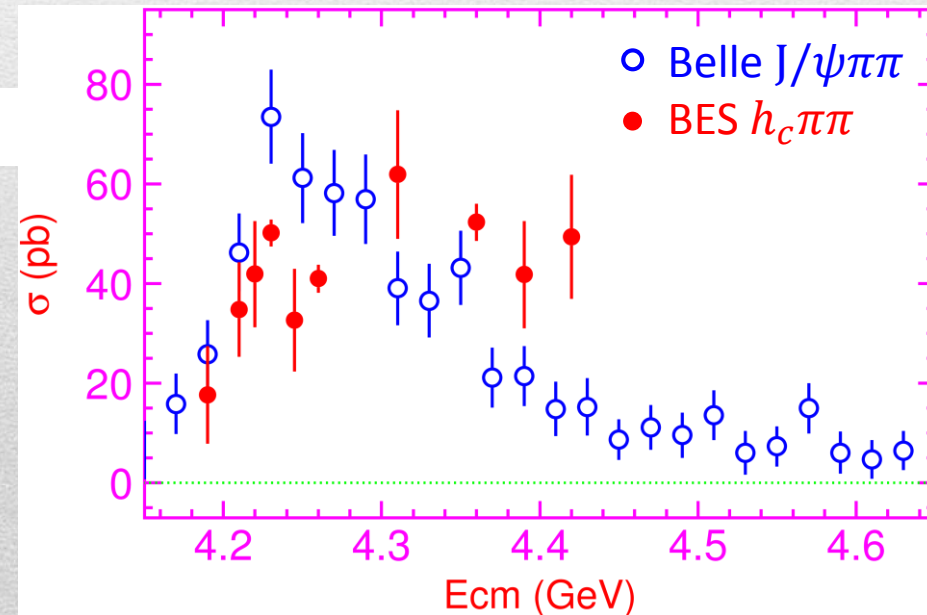
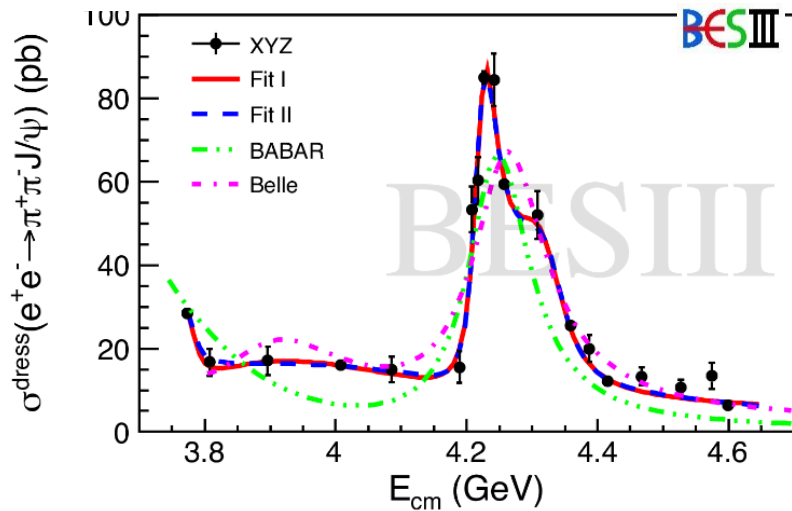
Vector Y states



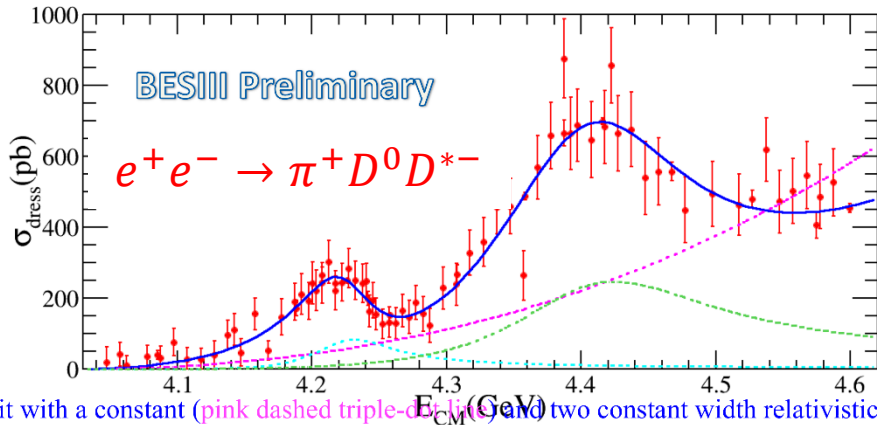
Lots of unexpected $J^{PC} = 1^{--}$ states found in ISR/direct production (and nowhere else!)
 Seen in few final states, mostly $J/\psi \pi\pi$ and $\psi(2S) \pi\pi$

Not seen decaying into open charm pairs
 Large HQSS violation

BESIII, PRL118, 092001 (2017) $e^+e^- \rightarrow J/\psi \pi\pi$



Vector Y states



Fit with a constant (pink dashed triple-dot line) and two constant width relativistic BW functions (green dashed double-dot line and aqua dashed line).

$$M(Y(4220)) = (4224.8 \pm 5.6 \pm 4.0) \text{ MeV}/c^2, \Gamma(Y(4220)) = (72.3 \pm 9.1 \pm 0.9) \text{ MeV}$$

$$M(Y(4390)) = (4400.1 \pm 9.3 \pm 2.1) \text{ MeV}/c^2, \Gamma(Y(4390)) = (181.7 \pm 16.9 \pm 7.4) \text{ MeV}$$

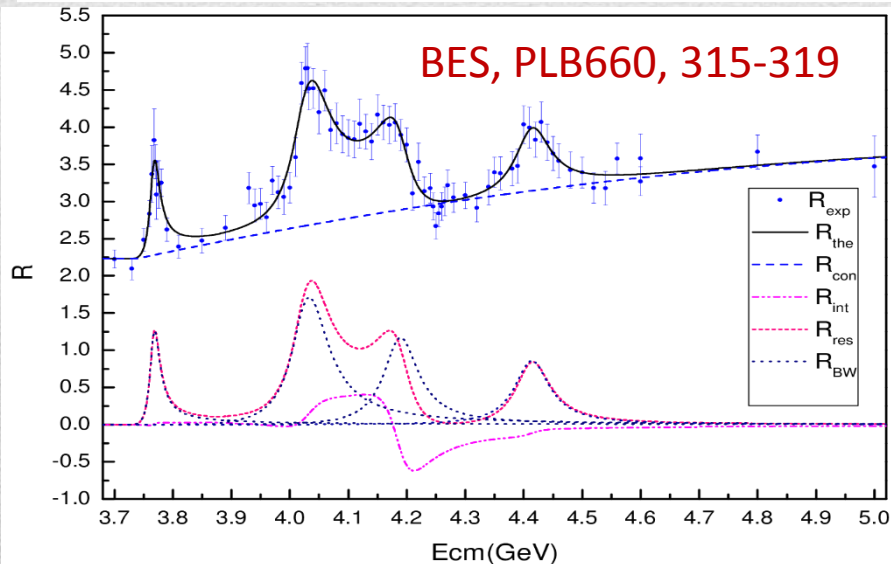
6

New BESIII data show a peculiar lineshape for the $Y(4260)$. The state appear lighter and narrower, compatible with the ones in $h_c \pi \pi$ and $\chi_{c0} \omega$. A broader old-fashioned $Y(4260)$ is appearing in $\bar{D} D^* \pi$, maybe indicating a $\bar{D} D_1$ dominance

Most of the information about the ordinary charmonium comes from a BES fit to the R ratio

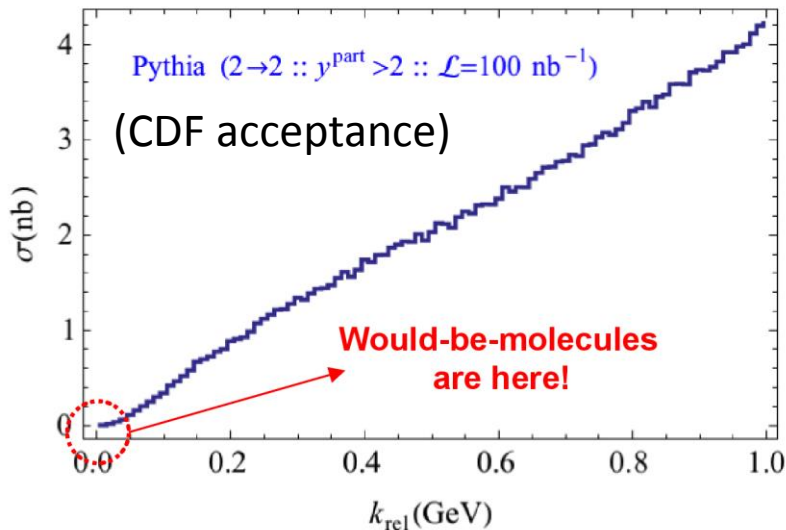
Times for a more refined analysis has come!

A. Amor, C. Fernandez-Ramirez, AP, U. Tamponi, in progress



Prompt production of $X(3872)$

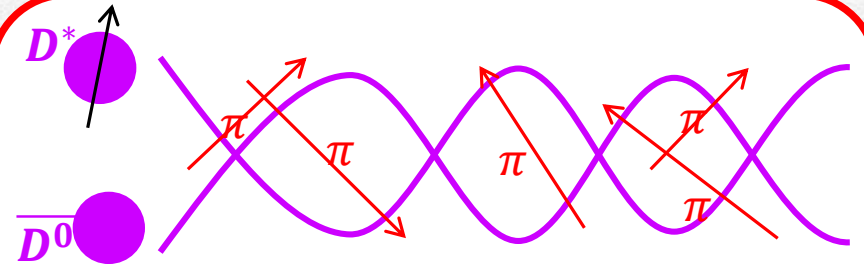
$X(3872)$ is the Queen of exotic resonances, the most popular interpretation is a $D^0\bar{D}^{0*}$ molecule (bound state, pole in the 1st Riemann sheet?) but it is copiously promptly produced at hadron colliders



$$\sigma_{MC}(p\bar{p} \rightarrow DD^* | k < k_{max}) \approx 0.1 \text{ nb}$$

$$\sigma_{exp}(p\bar{p} \rightarrow X(3872)) \approx 30 - 70 \text{ nb!!!}$$

Bignamini *et al.* PRL103 (2009) 162001



A solution can be FSI (rescattering of DD^*), which allow k_{max} to be as large as $5m_\pi$,
 $\sigma(p\bar{p} \rightarrow DD^* | k < k_{max}) \approx 230 \text{ nb}$

Artoisenet and Braaten, PRD81, 114018

However, the rescattering is flawed by the presence of pions that interfere with DD^* propagation. Estimating the effect of these pions increases σ , but not enough

Bignamini *et al.* PLB684, 228-230

Esposito, Piccinini, AP, Polosa, JMP 4, 1569

Guerrieri, Piccinini, AP, Polosa, PRD90, 034003

Production of $Y(4260)$ and $P_c(4450)$

Given the new lineshape by BESIII, we need to rethink the binding energy of the $Y(4260)$

J. Nys and AP, to appear

	Constituents	Bind. Energy	Bind. Mom.	Mediator
$X(3872)$	$\bar{D}^0 D^{*0}$	~ 100 keV	~ 50 MeV	1π (~ 300 MeV)
$Y(4260)$	$\bar{D} D_1$	~ 70 MeV	~ 400 MeV	2π (~ 600 MeV)
$P_c(4450)$	$\bar{D}^* \Sigma_c$	~ 10 MeV	~ 150 MeV	1π (~ 300 MeV)

If the states are purely hadron molecule, all the properties depend on the position of the pole with respect to threshold – all the features are universal

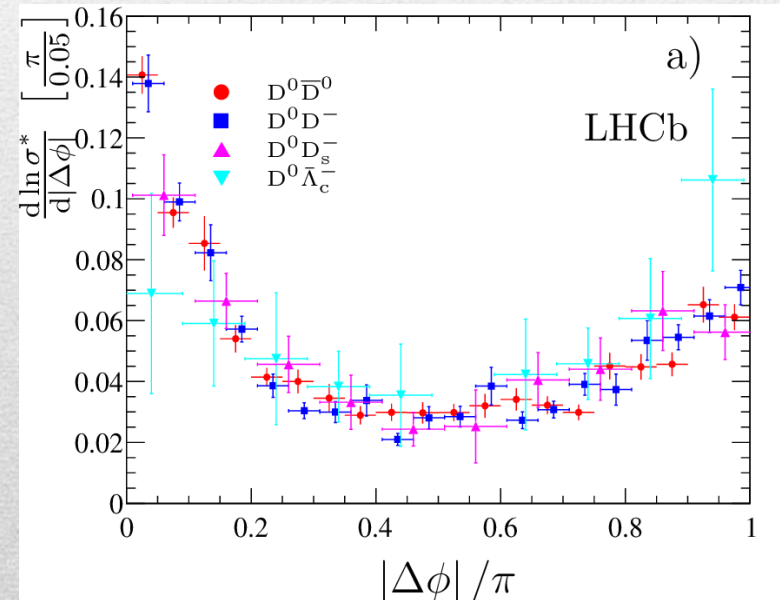
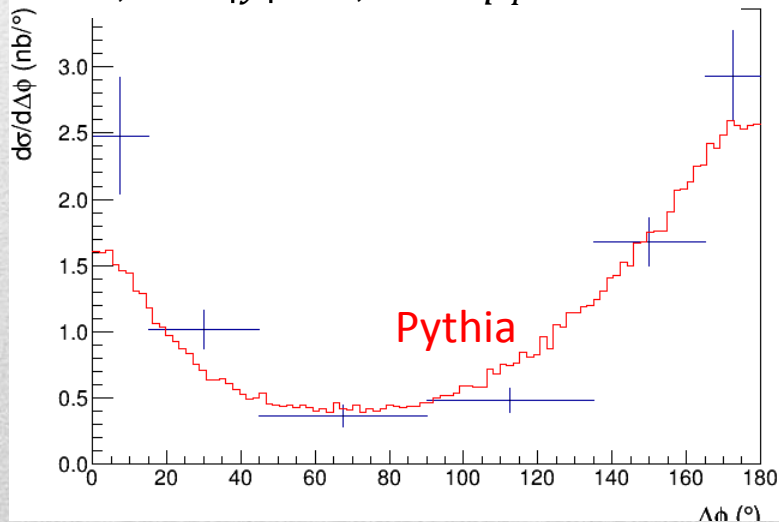
What does the production of $X(3872)$ implies for the other states?

Production of $Y(4260)$ and $P_c(4450)$

We can use Pythia to simulate the production of event, and calculate the relative production of $Y(4260)$ and $P_c(4450)$ with respect to the $X(3872)$ J. Nys and AP, to appear

We tune our MC on charm pair production For baryons we can double check with LHCb data

CDF data, $\sqrt{s} = 1.96$ TeV
 D^0, D^{*-} : $|y| < 1, 5.5 < p_T < 20$ GeV



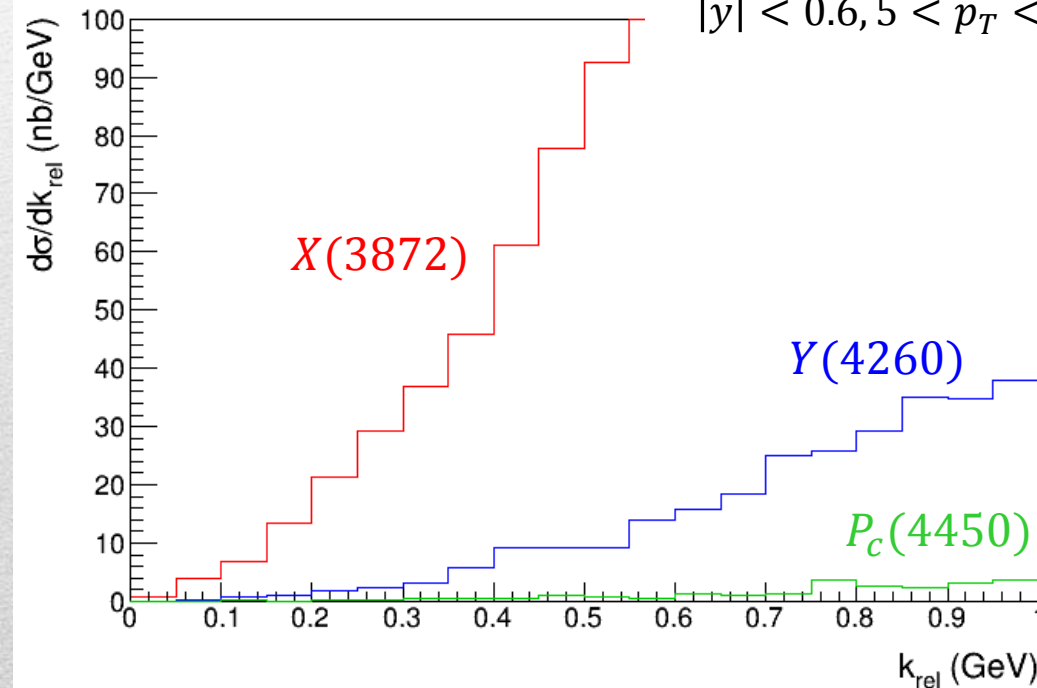
LHCb, $\sqrt{s} = 7$ TeV, **JHEP 1206, 141**
all: $2 < y < 4, 3 < p_T < 12$ GeV

Production of $Y(4260)$ and $P_c(4450)$

Naively, the fragmentation function of the D_1 is 1/10 of the D^* ,
but the cross section scales as k_{max}^3

J. Nys and AP, to appear

Pythia $p\bar{p}$, $\sqrt{s} = 1.96$ TeV
 $|y| < 0.6, 5 < p_T < 20$ GeV



	No FSI	With FSI
$Y(4260)/X$	23	0.75
$P_c(4450)/X$	1.0	0.01

The production of $Y(4260)$
is expected to be at worse comparable
with the $X(3872)$

Conclusions

- The discovery of **exotic states** has challenged the well established Charmonium framework
- Experiments are (too) prolific! **Constant feedback on predictions**
- Thorough **amplitude analyses** are requested to establish the existence of many of these states
- This also gives a guide to the interpretation of their **microscopic nature**

Thank you

BACKUP

Crossing symmetry in tensor formalisms

- ▶ The process $B \rightarrow \bar{D}\pi\pi$ is composed of scalar particles only

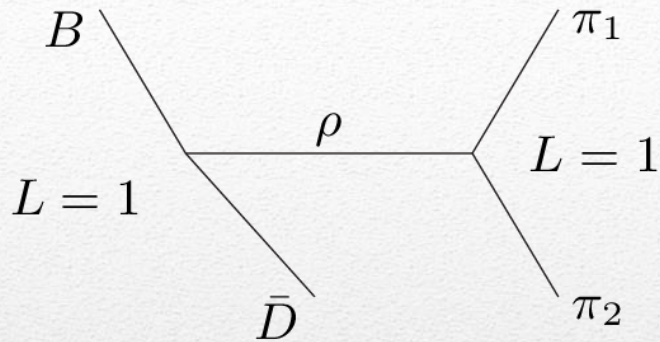
LHCb, PRD92, 032002 (2015)

- ▶ One defines the helicity angle in the isobar rest frame, then the amplitude is Lorentz Invariant
- ▶ Let's consider the ρ intermediate state, $B \rightarrow \bar{D}\rho(\rightarrow \pi\pi)$

$$A = \frac{m_B^2 + s - m_D^2}{2m_B^2} \cos\theta \times qp = \frac{E_\rho^{(B)}}{m_B} \cos\theta \times qp$$

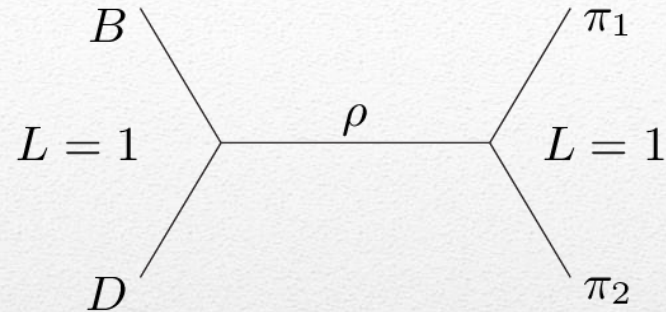
- ▶ The factors p and q are the $L = 1$ expected barrier factors. The additional factor is analytical in s , not a kinematical singularity. Why is it there?

Crossing symmetry in tensor formalisms



The tensor amplitude is given by $p_D^{(B)} \cdot p_\pi^{(\rho)}$, where $p_D^{(B)}$ is the breakup momentum **in the B frame**, and $p_\pi^{(\rho)}$ the decay momentum in the isobar frame

$$A = \frac{m_{B^0}^2 + s - m_{h_3}^2}{2m_{B^0}^2} pq \cos \theta$$



However, one can consider the scattering process just in the isobar rest frame.

$$A = pq \cos \theta$$

By crossing symmetry the amplitudes **must be** the same.

The usual implementation fails crossing symmetry

Note on $X(3872)$ production at hadron colliders and its molecular structure

Miguel Albaladejo,¹ Feng-Kun Guo,^{2,3} Christoph Hanhart,⁴

Ulf-G. Meißner,^{5,4} Juan Nieves,⁶ Andreas Nogga,⁴ and Zhi Yang⁵

¹*Departamento de Física, Universidad de Murcia, E-30071 Murcia, Spain*

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Universität Bonn, D-53115 Bonn, Germany

⁶*Instituto de Física Corpuscular (IFIC), Centro Mixto CSIC-UVA, Valencia, Spain*

Institutos de Investigación de Paterna, Aptd. 22085, E-46071 Valencia, Spain



The production of the $X(3872)$ as a hadronic molecule in hadron colliders is clarified. We show that the conclusion of Bignamini *et al.*, Phys. Rev. Lett. **103** (2009) 162001, that the production of the $X(3872)$ at high p_T implies a non-molecular structure, does not hold. In particular, using the well understood properties of the deuteron wave function as an example, we identify the relevant scales in the production process.

The argument is about the value of a nonnormalizable wave function. Any argument about where the wave function is localized must be calculated for the modulus square

A widespread argument against the interpretation of the $X(3872)$ as a hadronic molecule is its copious production at hadron colliders. Based on the inequality¹

$$\sigma(\bar{p}p \rightarrow X) \sim \left| \int d^3\mathbf{k} \langle X | D^0 \bar{D}^{*0}(\mathbf{k}) \rangle \langle D^0 \bar{D}^{*0}(\mathbf{k}) | \bar{p}p \rangle \right|^2$$

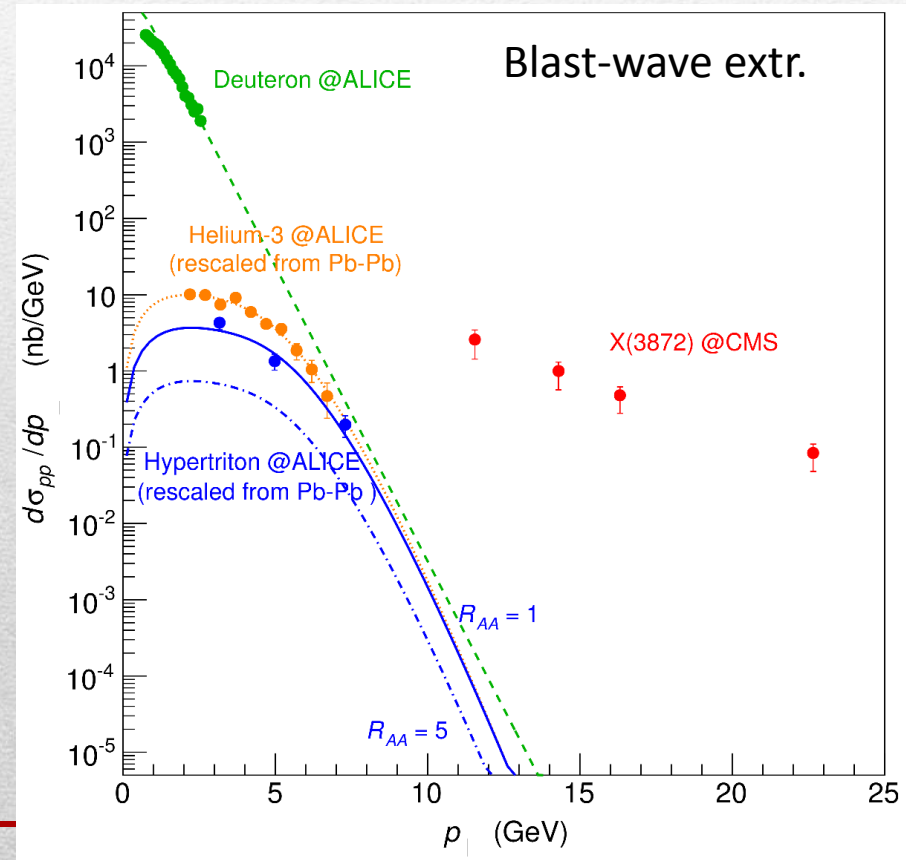
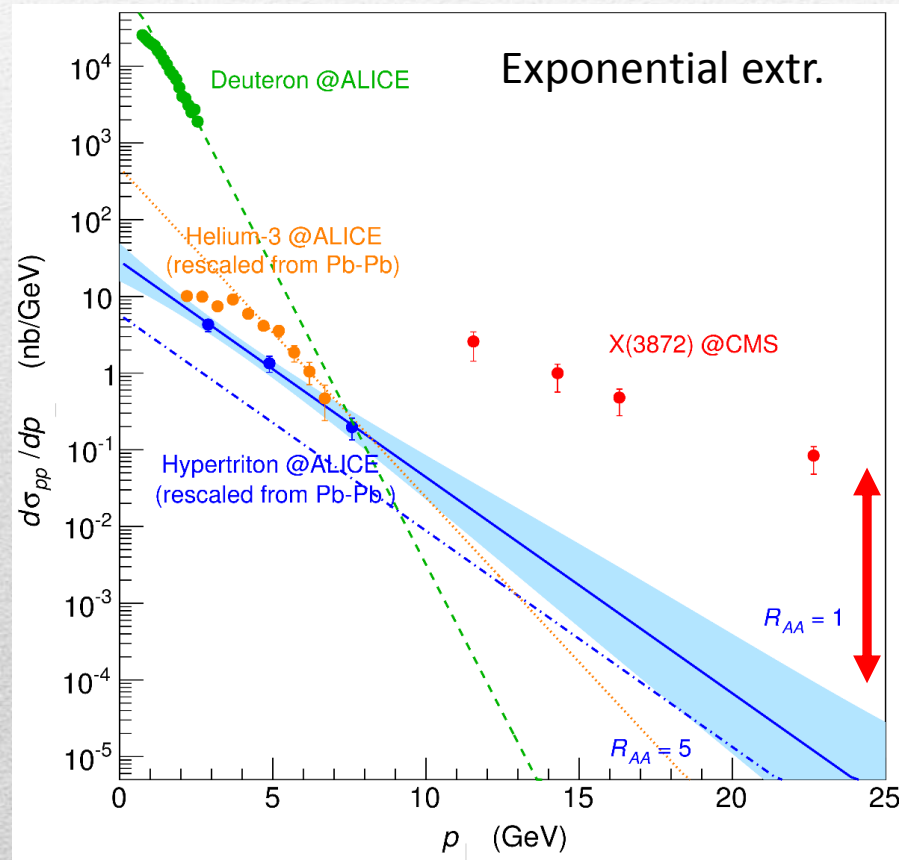
v:1709.09101v1 [hep-ph] 26 Sep 2017

Light nuclei at ALICE vs. $X(3872)$

Esposito, Guerrieri, Maiani, Piccinini, AP, Polosa, Riquer, PRD92, 034028

We assume a pure Glauber model ($R_{AA} = 1$) and a value $R_{AA} = 5$ to rescale Pb-Pb data to pp

The $X(3872)$ is way larger than the extrapolated cross section

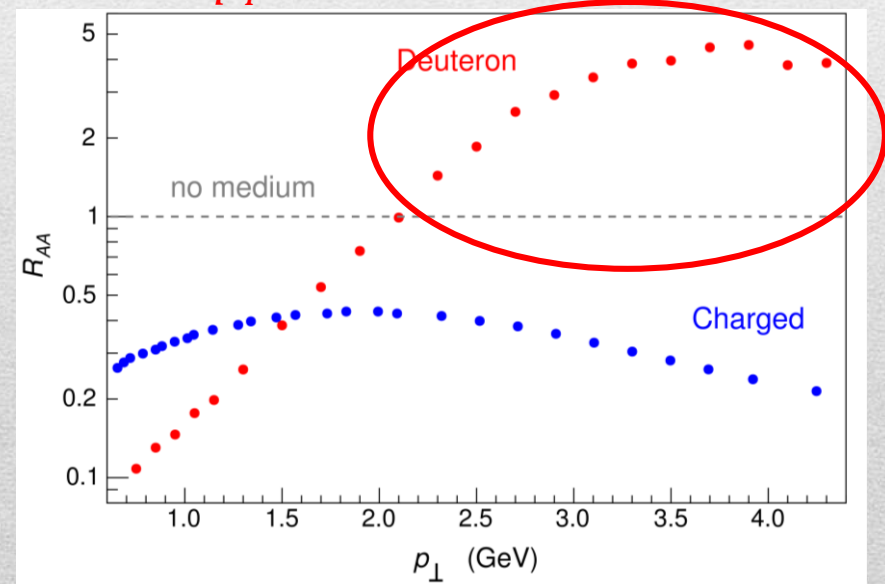
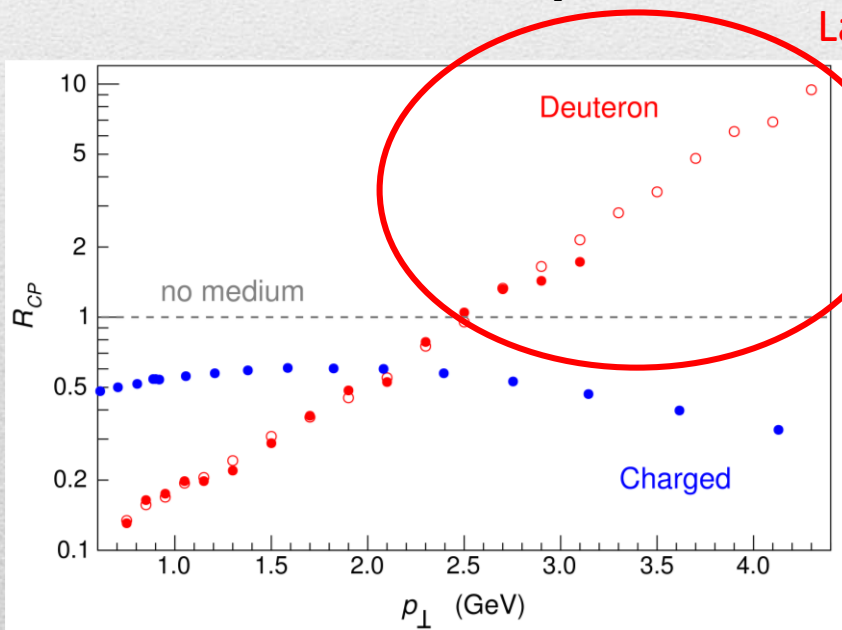


Nuclear modification factors

We can use deuteron data to extract the values of the nuclear modification factors (caveat: for RAA data have different \sqrt{s})

$$R_{CP} = \frac{N_{coll}^P \left(\frac{dN}{dp_T} \right)_C}{N_{coll}^C \left(\frac{dN}{dp_T} \right)_P}$$

$$R_{AA} = \frac{\left(\frac{dN}{dp_T} \right)_{Pb-Pb}}{N_{coll} \left(\frac{dN}{dp_T} \right)_{pp}}$$

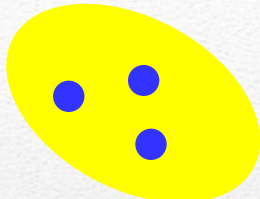


Hadron Spectroscopy

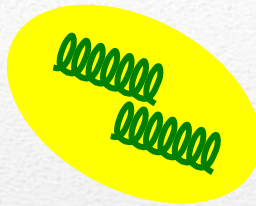
Meson



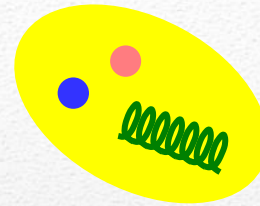
Baryon



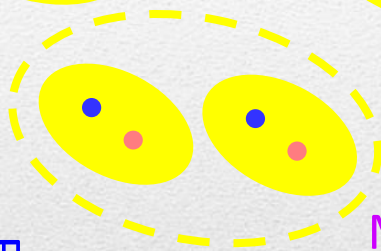
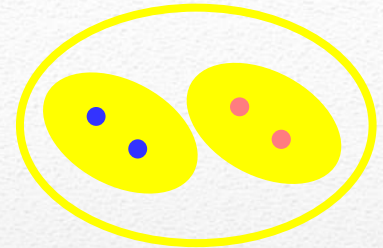
Glueball



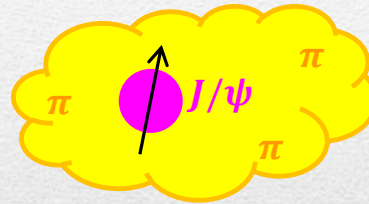
Hybrids



Tetraquark



Molecule



Hadroquarkonium

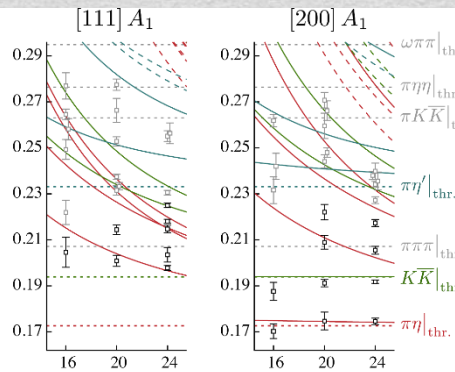


Experiment

Lattice QCD

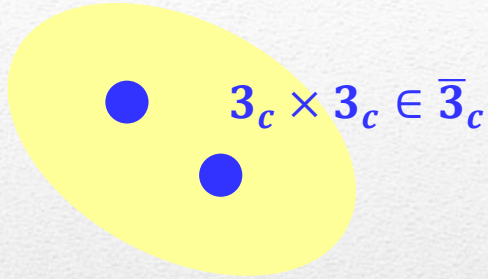


Interpretations on the spectrum leads to understanding fundamental laws of nature



Diquarks

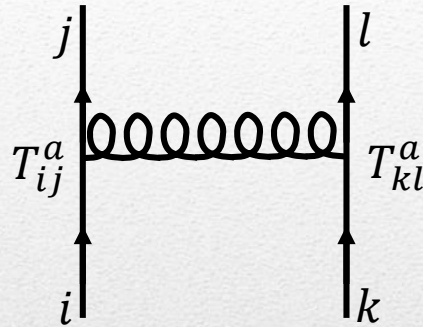
Attraction and repulsion in 1-gluon exchange approximation is given by



$$R = \frac{1}{2} (C_2(R_{12}) - C_2(R_1) - C_2(R_2))$$

$$R_1 = -\frac{4}{3}, R_8 = +\frac{1}{6}$$

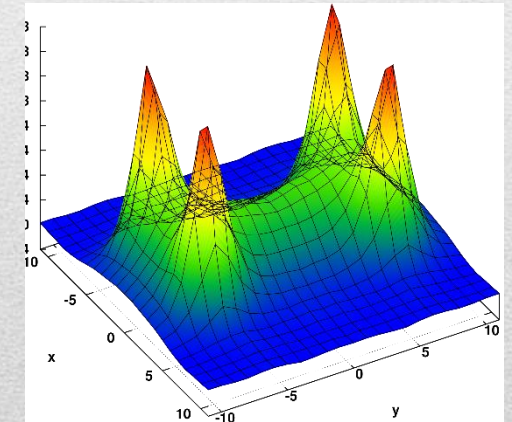
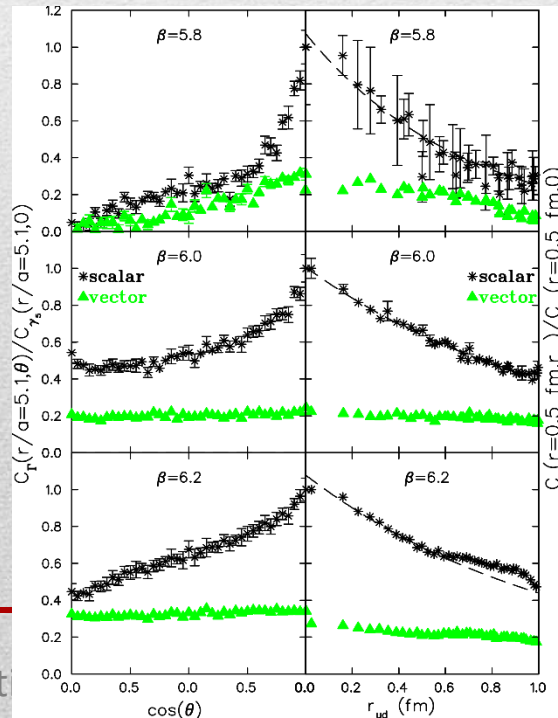
$$R_3 = -\frac{2}{3}, R_6 = +\frac{1}{3}$$



The singlet $\mathbf{1}_c$ is attractive

A diquark in $\bar{\mathbf{3}}_c$ is attractive

Evidence (?) of diquarks in LQCD,
Alexandrou, de Forcrand, Lucini,
PRL 97, 222002



H-shape with a 4 quark system
Cardoso, Cardoso, Bicudo,
PRD84, 054508

Tetraquark

In a constituent (di)quark model, we can think of a **diquark-antidiquark compact state**

$$[cq]_{S=0}[\bar{c}\bar{q}]_{S=1} + h.c.$$

Maiani, Piccinini, Polosa, Riquer PRD71 014028

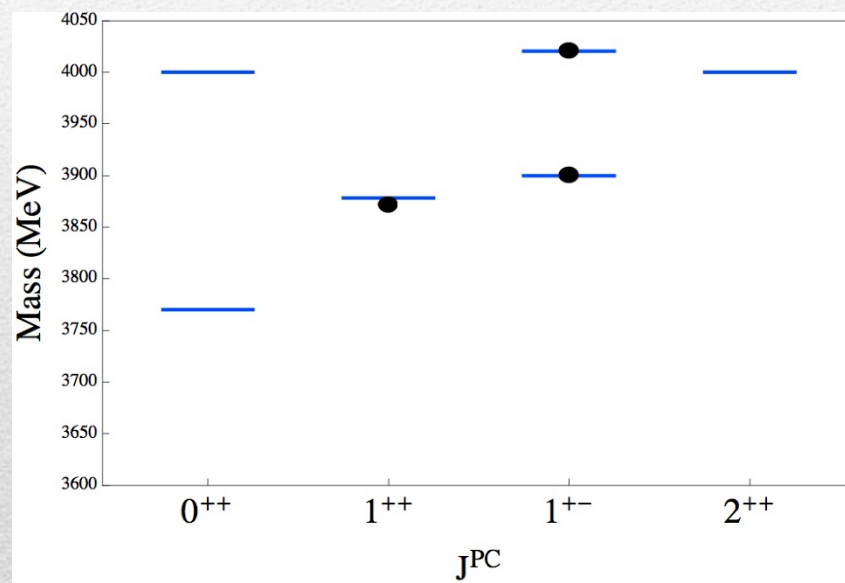
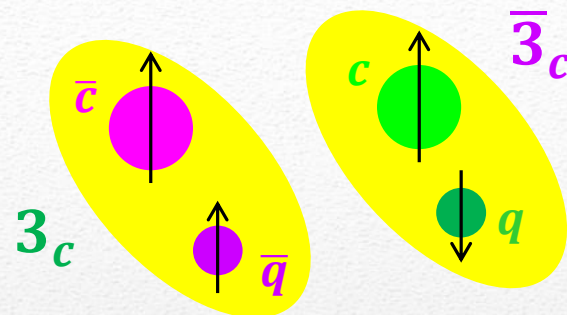
Faccini, Maiani, Piccinini, AP, Polosa, Riquer PRD87 111102

Maiani, Piccinini, Polosa, Riquer PRD89 114010

Spectrum according to **color-spin hamiltonian**
(all the terms of the Breit-Fermi hamiltonian are absorbed into a constant diquark mass):

$$H = \sum_{dq} m_{dq} + 2 \sum_{i<j} \kappa_{ij} \vec{S}_i \cdot \vec{S}_j \frac{\lambda_i^a}{2} \frac{\lambda_j^a}{2}$$

- Decay pattern mostly driven by **HQSS** ✓
- Fair understanding of existing spectrum ✓
- A full nonet for each level is expected ✗



New ansatz: the diquarks are compact objects spatially separated from each other,
only $\kappa_{cq} \neq 0$

Existing spectrum is fitted if $\kappa_{cq} = 67$ MeV

Tetraquark

Maiani, Piccinini, Polosa, Riquer PRD89 114010

J^{PC}	$cq \bar{c}\bar{q}$	$c\bar{c} q\bar{q}$	Resonance Assig.	Decays
0^{++}	$ 0, 0\rangle$	$1/2 0, 0\rangle + \sqrt{3}/2 1, 1\rangle_0$	$X_0(\sim 3770 \text{ MeV})$	$\eta_c, J/\psi + \text{light mesons}$
0^{++}	$ 1, 1\rangle_0$	$\sqrt{3}/2 0, 0\rangle - 1/2 1, 1\rangle_0$	$X'_0(\sim 4000 \text{ MeV})$	$\eta_c, J/\psi + \text{light mesons}$
1^{++}	$1/\sqrt{2}(1, 0\rangle + 0, 1\rangle)$	$ 1, 1\rangle_1$	$X_1 = X(3872)$	$J/\psi + \rho/\omega, DD^*$
1^{+-}	$1/\sqrt{2}(1, 0\rangle - 0, 1\rangle)$	$1/\sqrt{2}(1, 0\rangle - 0, 1\rangle)$	$Z = Z(3900)$	$J/\psi + \pi, h_c/\eta_c + \pi/\rho$
1^{+-}	$ 1, 1\rangle_1$	$1/\sqrt{2}(1, 0\rangle + 0, 1\rangle)$	$Z' = Z(4020)$	$J/\psi + \pi, h_c/\eta_c + \pi/\rho$
2^{++}	$ 1, 1\rangle_2$	$ 1, 1\rangle_2$	$X_2(\sim 4000 \text{ MeV})$	$J/\psi + \text{light mesons}$

$$H_{\text{eff}} = 2m_Q + \frac{B_Q}{2} \mathbf{L}^2 - 3\kappa_{cq} + 2a_Y \mathbf{L} \cdot \mathbf{S} + b_Y \frac{\langle S_{12} \rangle}{4} + \kappa_{cq} [2(\mathbf{S}_q \cdot \mathbf{S}_c + \mathbf{S}_{\bar{q}} \cdot \mathbf{S}_{\bar{c}}) + 3]$$

Ali, Maiani, *et al.* arXiv:1708.04650

Two different mass scenarios

$$M_1 = 4008 \pm 40_{-28}^{+114}, \quad M_2 = 4230 \pm 8, \\ M_3 = 4341 \pm 8, \quad M_4 = 4643 \pm 9.$$

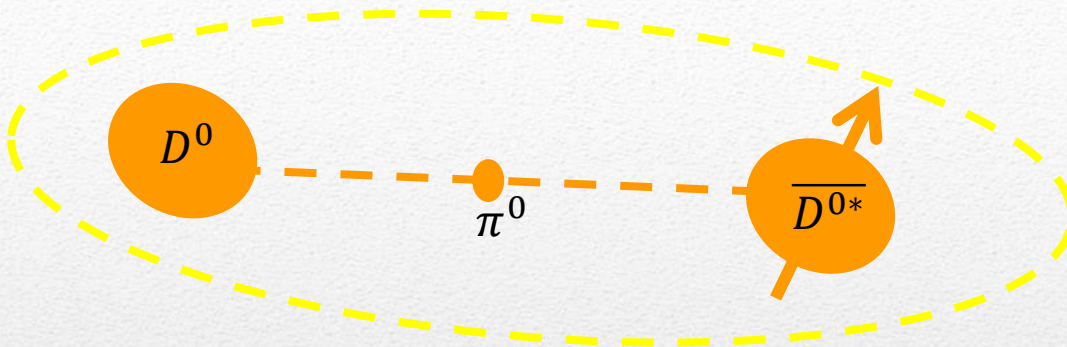
$$M_1 = 4219.6 \pm 3.3 \pm 5.1, \quad M_2 = 4333.2 \pm 19.9, \\ M_3 = 4391.5 \pm 6.3, \quad M_4 = 4643 \pm 9,$$

Prediction for a high Y_5

$$M_5 = \begin{cases} 6539 \text{ MeV} & \text{SI(c1)} \\ 6589 \text{ MeV} & \text{SI(c2)} \\ 6862 \text{ MeV} & \text{SII(c1)} \\ 6899 \text{ MeV} & \text{SII(c2)} \end{cases}$$

Label	$ S_Q, S_{\bar{Q}}; S, L\rangle_J$
Y_1	$ 0, 0; 0, 1\rangle_1$
Y_2	$(1, 0; 1, 1\rangle_1 + 0, 1; 1, 1\rangle_1)/\sqrt{2}$
Y_3	$ 1, 1; 0, 1\rangle_1$
Y_4	$ 1, 1; 2, 1\rangle_1$
Y_5	$ 1, 1; 2, 3\rangle_1$

Other models: Molecule



Tornqvist, Z.Phys. C61, 525
 Braaten and Kusunoki, PRD69 074005
 Swanson, Phys.Rept. 429 243-305

$$\begin{aligned}
 X(3872) &\sim \bar{D}^0 D^{*0} \\
 Z_c(3900) &\sim \bar{D}^0 D^{*+} \\
 Z'_c(4020) &\sim \bar{D}^{*0} D^{*+} \\
 Y(4260) &\sim \bar{D} D_1
 \end{aligned}$$

A **deuteron-like meson pair**, the interaction is mediated by the exchange of light mesons

- Some model-independent relations (**Weinberg's theorem**) ✓
- Good description of **decay patterns** (mostly to constituents) and X(3872) **isospin violation** ✓
- States appear **close to thresholds** ✓ (but **Z(4430)** ✗)
- Lifetime of constituents has to be $\gg 1/m_\pi$
- Binding energy varies from -70 to -0.1 MeV, or even **positive** (repulsive interaction) ✗
- **Unclear spectrum** (a state for each threshold?) – **depends on potential models** ✗

$$V_\pi(r) = \frac{g_{\pi N}^2}{3} (\vec{\tau}_1 \cdot \vec{\tau}_2) \left\{ [3(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_2 \cdot \hat{r}) - (\vec{\sigma}_1 \cdot \vec{\sigma}_2)] \left(1 + \frac{3}{(m_\pi r)^2} + \frac{3}{m_\pi r} \right) + (\vec{\sigma}_1 \cdot \vec{\sigma}_2) \right\} \frac{e^{-m_\pi r}}{r}$$

Needs regularization, cutoff dependence

Multiscale system

Systematically integrate out the heavy scale,

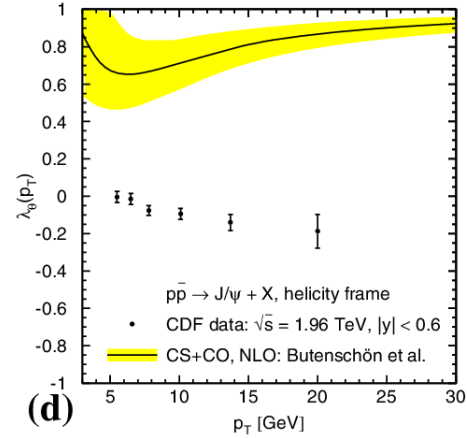
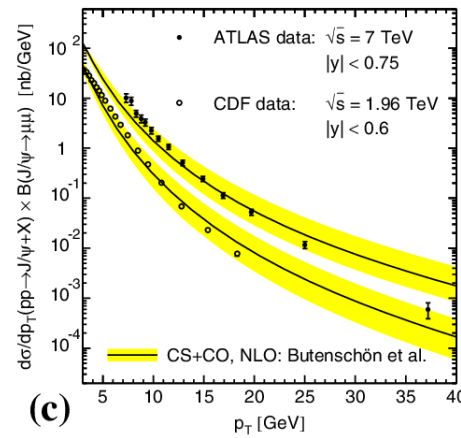
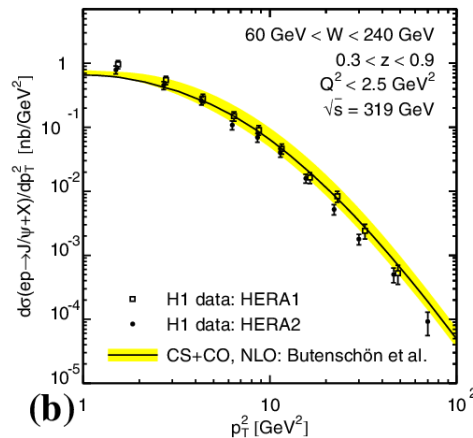
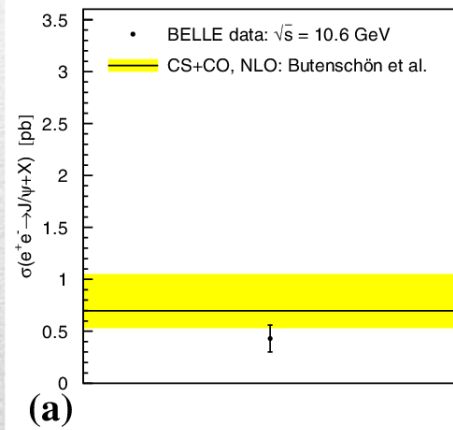
$$m_Q \gg \Lambda_{QCD}$$

$$m_Q \gg m_Q v \gg m_Q v^2$$

Full QCD \longrightarrow NRQCD \longrightarrow pNRQCD

$$m_b \sim 5 \text{ GeV}, m_c \sim 1.5 \text{ GeV}$$

$$v_b^2 \sim 0.1, v_c^2 \sim 0.3$$

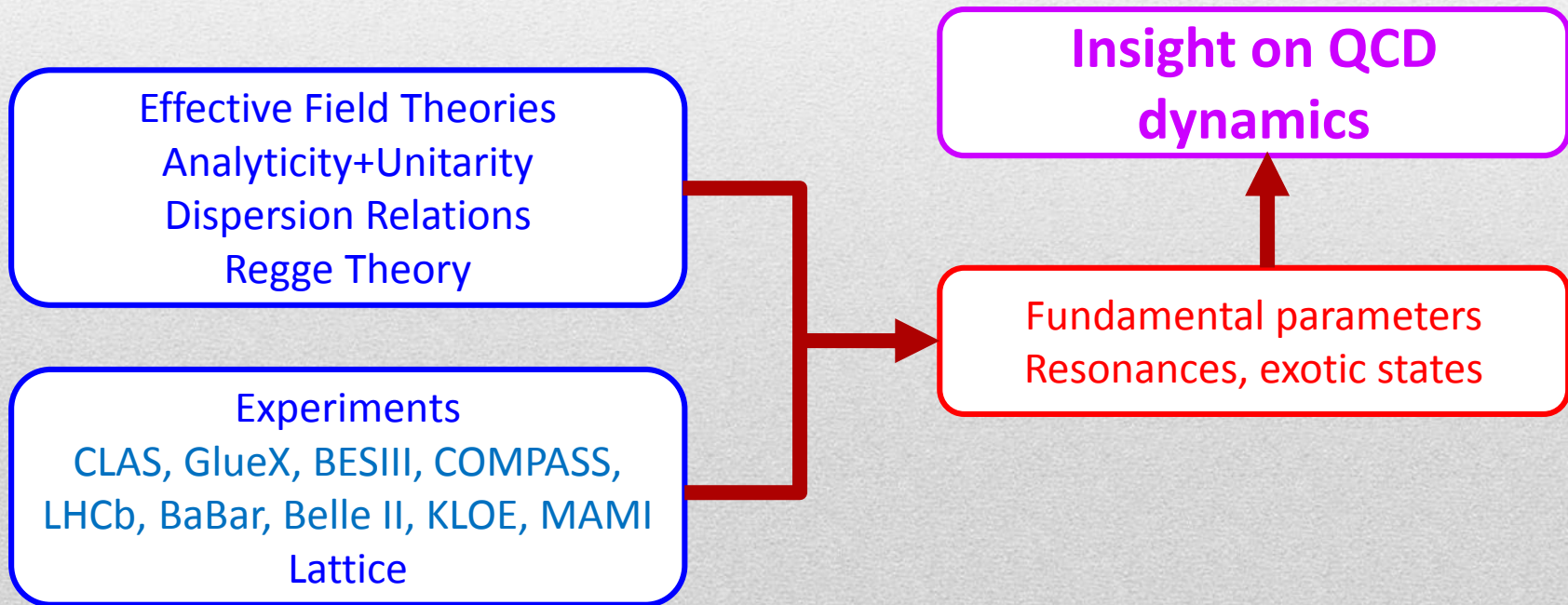


Factorization (to be proved)
of universal LDMEs

Good description of many production channels,
some known puzzles (polarizations)

Joint Physics Analysis Center

- **Joint effort** between **theorists** and **experimentalists** to work together to make the best use of the next generation of very precise data taken at JLab and in the world
- Created in 2013 by JLab & IU agreement
- It is engaged in **education** of further generations of hadron physics practitioners



Three-Body Unitarity

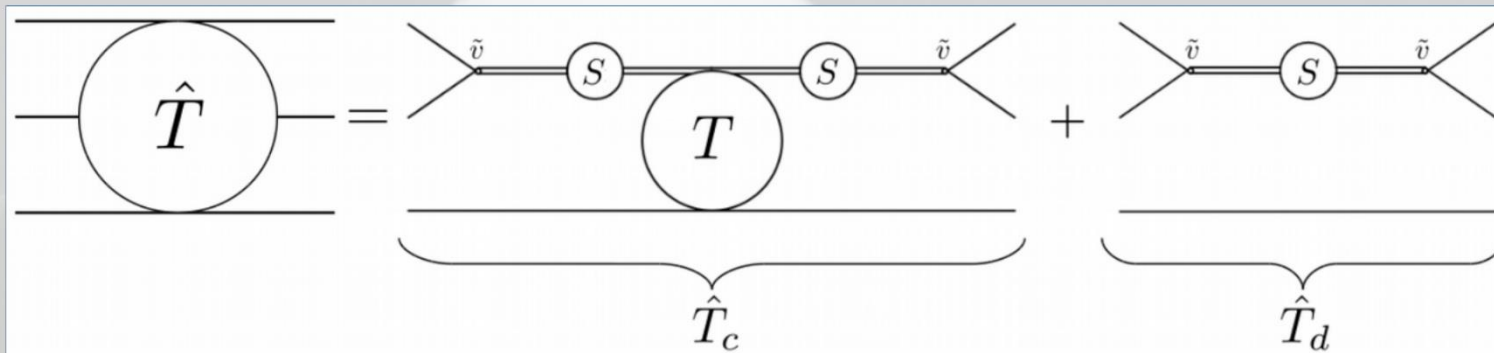
Mai, Hu, Doring, AP, Szczepaniak, EPJA53, 9, 177

AAY(1968)

Original study by Amado/Aaron/Young

- 3-dimensional integral equation from unitarity constraint & BSE ansatz
- valid below break-up energies ($E < 3m$)
- analyticity constraints unclear

One has to begin with asymptotic states



- v a general but cut-free (in the phys. region) function
- two-body interaction is parametrized by an “isobar”

= has definite QN and correct r.h.-singularities w.r.t invariant mass

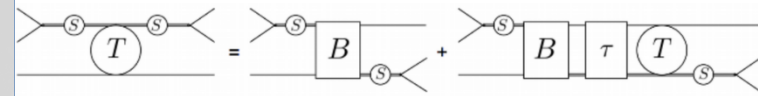
- S and T are yet unknown functions

M. Mai

Three-Body Unitarity

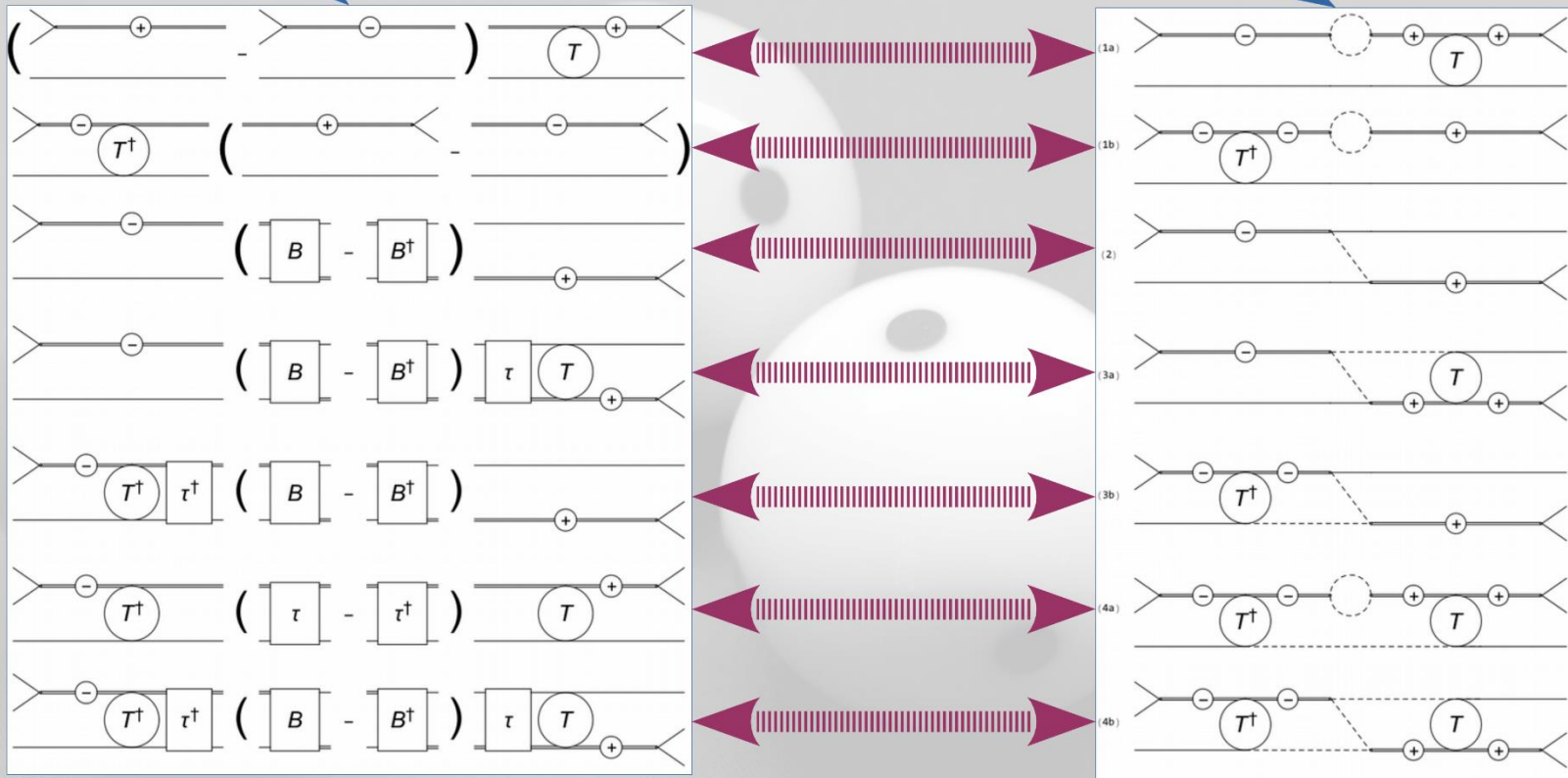
A general ansatz for the Isobar-spectator interaction

→ B & τ are unknown!!!



3-body Unitarity (normalization condition ↔ phase space integral)

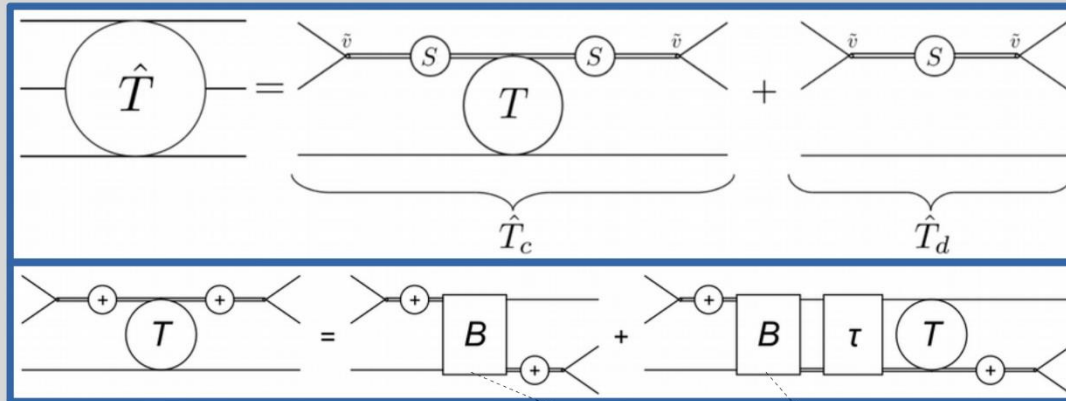
$$\langle q_1, q_2, q_3 | (\hat{T} - \hat{T}^\dagger) | p_1, p_2, p_3 \rangle = i \int_P \langle q_1, q_2, q_3 | \hat{T}^\dagger | k_1, k_2, k_3 \rangle \langle k_1, k_2, k_3 | \hat{T} | p_1, p_2, p_3 \rangle$$



M. Mai

SCATTERING AMPLITUDE

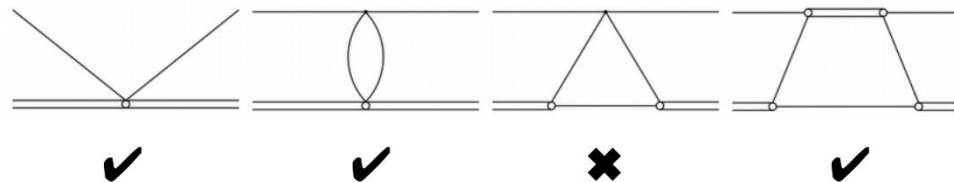
3 → 3 scattering amplitude is a 3-dimensional integral equation



- Imaginary parts (B , τ , S) are fixed by **unitarity/matching**
For simplicity $v=\lambda$ (full relations available)

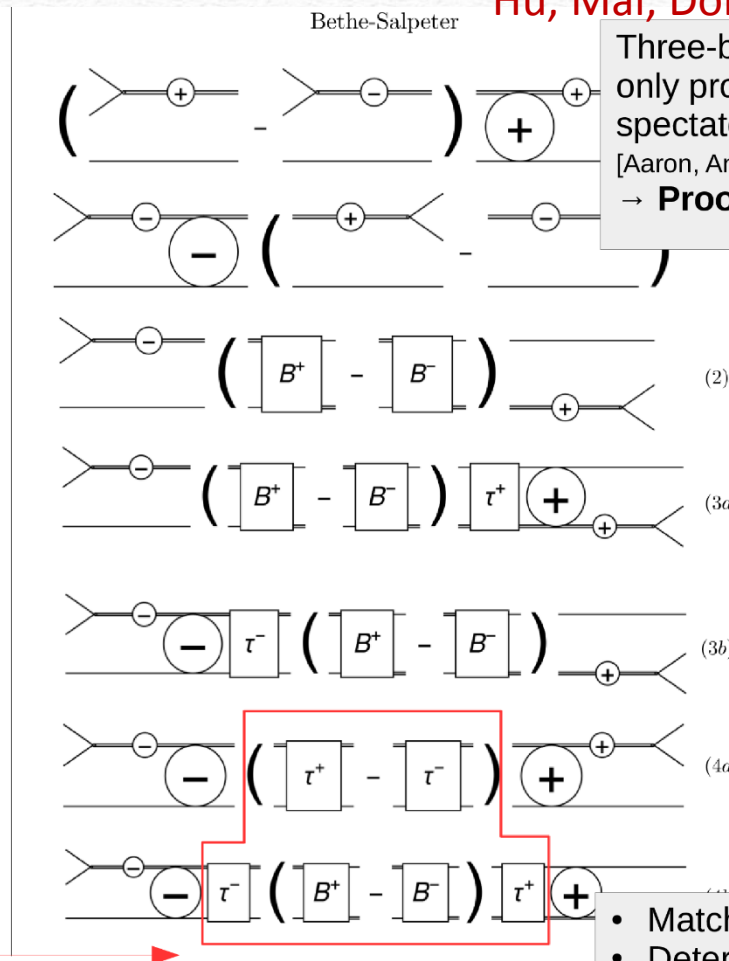
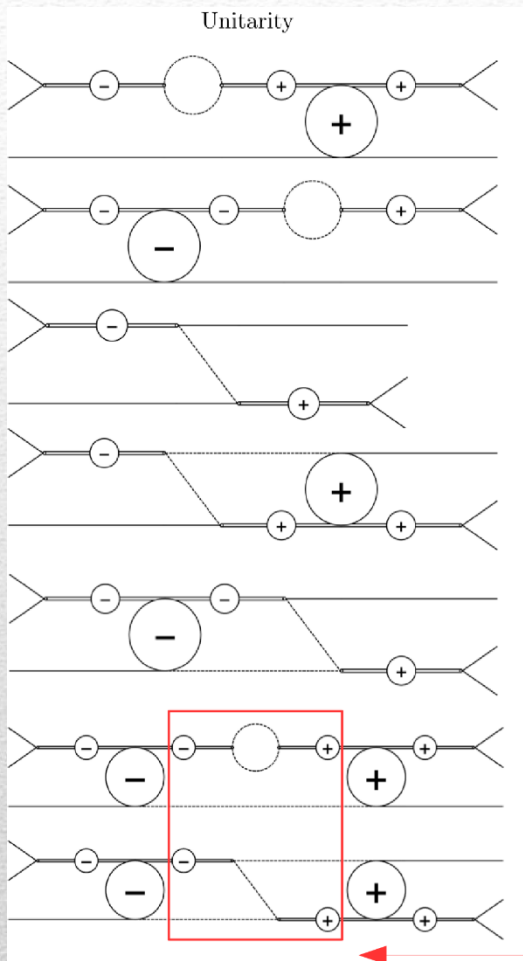
$$\langle q|B(s)|p\rangle = -\frac{\lambda^2}{2\sqrt{m^2+Q^2}\left(E_Q - \sqrt{m^2+Q^2} + i\epsilon\right)}$$

- un-subtracted dispersion relation
- one- π exchange in TOPT
- real contributions can be added to B



Three-Body Unitarity

Hu, Mai, Doring, AP, Szczepaniak, in prep.



Three-body unitarity for isobars only proven for bound state-spectator scattering [Aaron, Amamdo, Young, PR (1969)]
 → **Proof above breakup needed!**

Bound-state particle scattering requires only comparing these.

- Match Ansatz to unitarity
- Determine three-body amplitude
- Consistency of matching relations shown.
- Proof finished

Joint Physics Analysis Center



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V. Mathieu, V. Pauk, A. Pilloni,
V. Mokeev (JLab)

P. Guo (Cal. State U.)

J. Castro, C. Fernandez-Ramirez (UNAM)



L. Bibzrycki, R. Kaminski
(Krakow)

J. Nys (Ghent U.)

M. Mikhasenko (Bonn U.)

L. Dai (FZ Julich)

I. Danilkin,

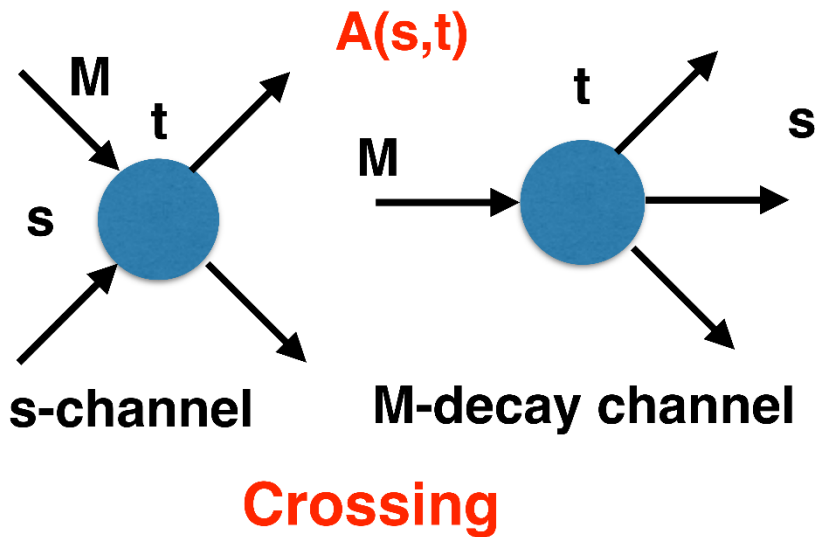
A. Hiller Blin (Mainz U.)

A. Celentano (INFN-GE)

M. Albaladejo (Valencia U.)

Students, Postdocs, Faculties

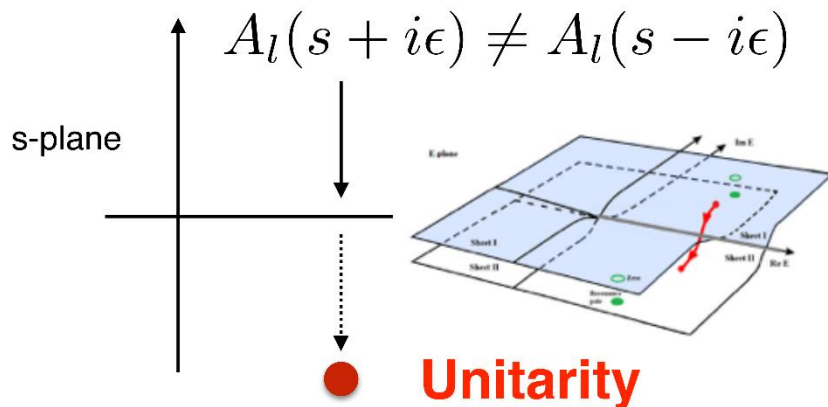
S-Matrix principles



$$A(s, t) = \sum_l A_l(s) P_l(z_s)$$

Analyticity

$$A_l(s) = \lim_{\epsilon \rightarrow 0} A_l(s + i\epsilon)$$

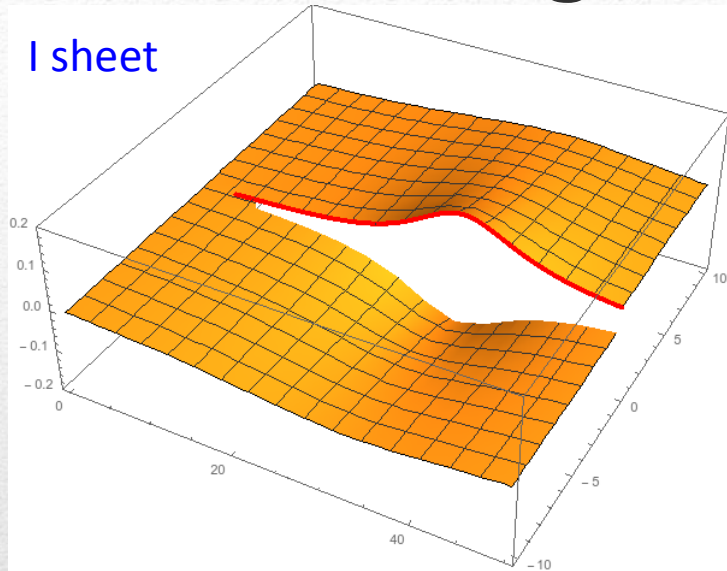


These are constraints the amplitudes have to satisfy, but do not fix the dynamics

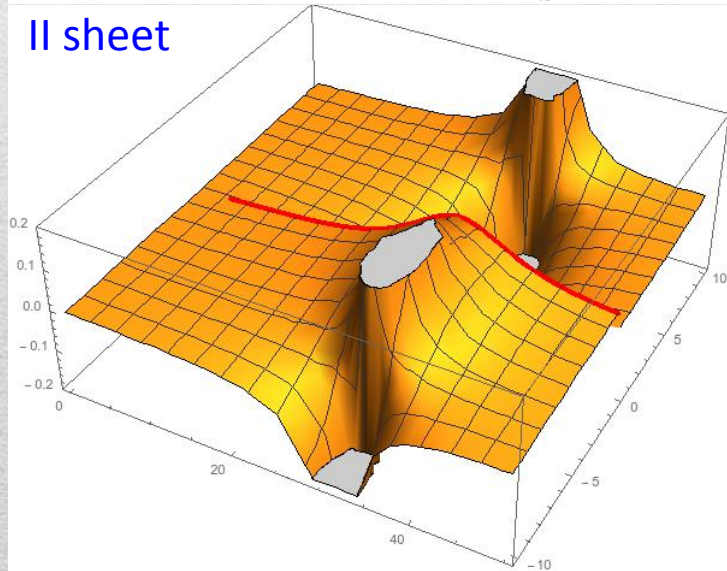
Resonances (QCD states) are poles in the unphysical Riemann sheets

Pole hunting

I sheet

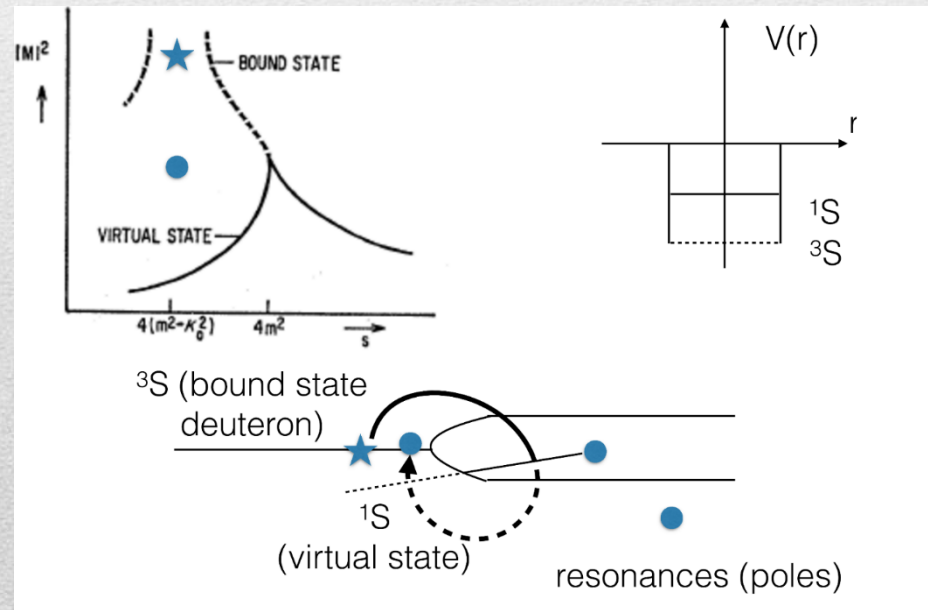


II sheet



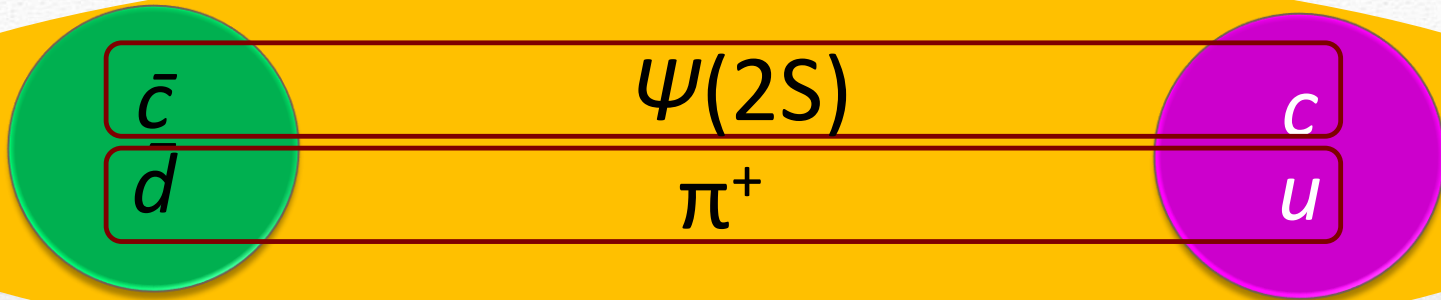
Bound states on the real axis 1st sheet

Not-so-bound (virtual) states on the real axis 2nd sheet



Dynamical movie

$Z^+(4430)$



Brodsky, Hwang, Lebed PRL 113 112001

- Since this is still a $\mathbf{3} \leftrightarrow \bar{\mathbf{3}}$ color interaction, just use the Cornell potential:

$$V(r) = -\frac{4}{3} \frac{\alpha_s}{r} + br + \frac{32\pi\alpha_s}{9m_{cq}^2} \left(\frac{\sigma}{\sqrt{\pi}}\right)^3 e^{-\sigma^2 r^2} \mathbf{S}_{cq} \cdot \mathbf{S}_{\bar{c}\bar{q}},$$

e.g. Barnes *et al.*, PRD 72, 054026

- Use that the kinetic energy released in $\bar{B}^0 \rightarrow K^- Z^+(4430)$ converts into potential energy until the diquarks come to rest
- Hadronization most effective at this point (WKB turning point)

$$r_Z = 1.16 \text{ fm}, \langle r_{\psi(2S)} \rangle = 0.80 \text{ fm}, \langle r_{J/\psi} \rangle = 0.39 \text{ fm}$$

$$\frac{B(Z^+(4430) \rightarrow \psi(2S)\pi^+)}{B(Z^+(4430) \rightarrow J/\psi \pi^+)} \sim 72$$

($> 10 \text{ exp.}$)

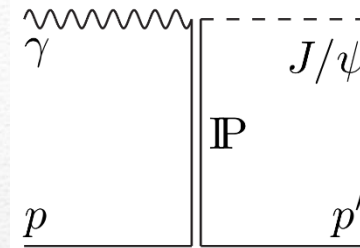
Pentaquark photoproduction

To exclude any rescattering mechanism, we propose to search the $P_c(4450)$ state in **photoproduction**

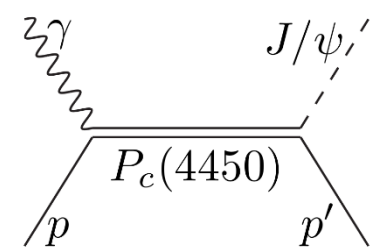
We use the (few) existing data and **VMD + pomeron inspired bkg** to estimate the cross section

$$J^P = (3/2)^-$$

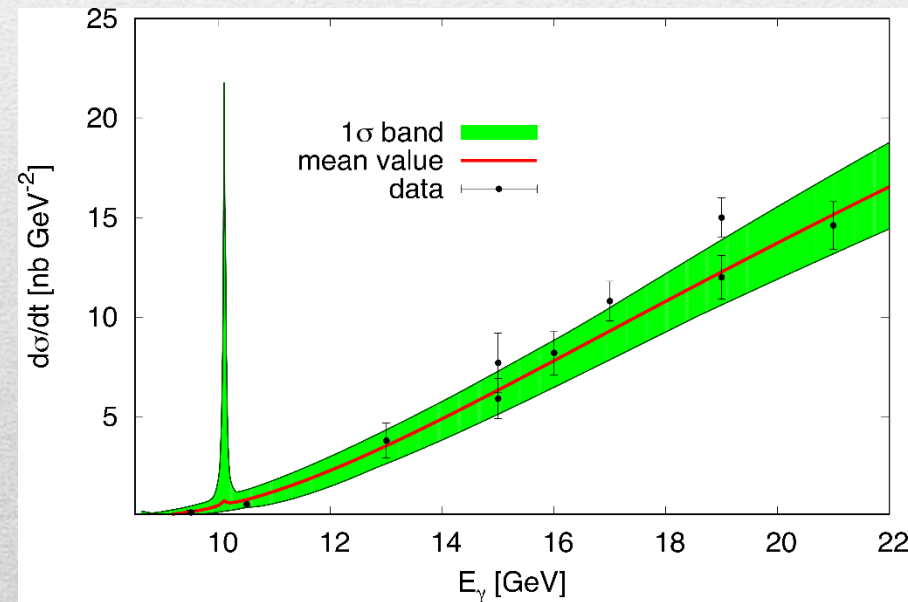
σ_s (MeV)	0	60	120
A	$0.156^{+0.029}_{-0.020}$	$0.157^{+0.039}_{-0.021}$	$0.157^{+0.037}_{-0.022}$
α_0	$1.151^{+0.018}_{-0.020}$	$1.150^{+0.018}_{-0.026}$	$1.150^{+0.015}_{-0.023}$
α' (GeV ⁻²)	$0.112^{+0.033}_{-0.054}$	$0.111^{+0.037}_{-0.064}$	$0.111^{+0.038}_{-0.054}$
s_t (GeV ²)	$16.8^{+1.7}_{-0.9}$	$16.9^{+2.0}_{-1.6}$	$16.9^{+2.0}_{-1.1}$
b_0 (GeV ⁻²)	$1.01^{+0.47}_{-0.29}$	$1.02^{+0.61}_{-0.32}$	$1.03^{+0.49}_{-0.31}$
$\mathcal{B}_{\psi p}$ (95% CL)	$\leq 29\%$	$\leq 30\%$	$\leq 23\%$



(a) Pomeron exchange



(b) Resonant contribution



A. Blin, AP et al. (JPAC), PRD94, 034002

$Z_c(3900) \rightarrow \eta_c \rho$

Esposito, Guerrieri, AP, PLB 746, 194-201

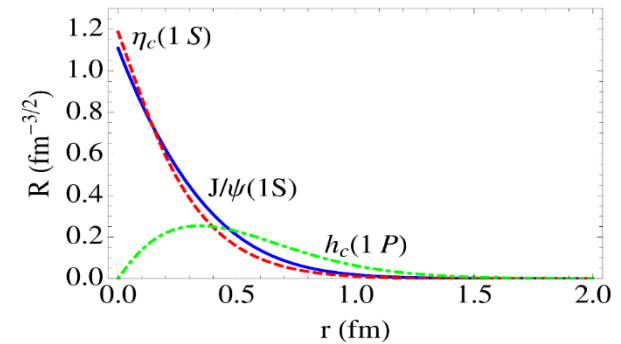
If tetraquark

Kinematics with HQSS, dynamics estimated according to Brodsky et al., PRL113, 112001

$$A = \langle \chi_{c\bar{c}} | \chi_c \otimes \chi_{\bar{c}} \rangle \langle \phi_{c\bar{c}} | \hat{T}_{\perp HQS} | \phi[cq][\bar{c}\bar{q}] \rangle + \mathcal{O}\left(\frac{\Lambda_{QCD}}{m_c}\right)$$

Clebsch-Gordan

Uncertainty
~ 25%



Reduced matrix element

- approximated as a constant
- or $\propto \psi_{c\bar{c}}(r_Z)$

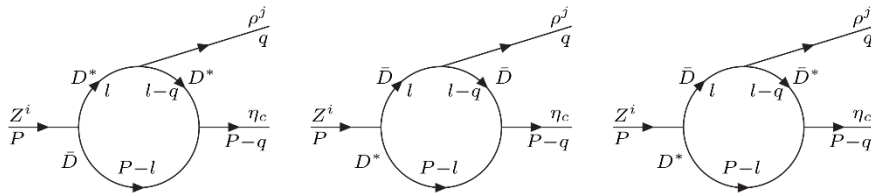
	Kinematics only		Dynamics included	
	type I	type II	type I	type II
$\frac{\mathcal{BR}(Z_c \rightarrow \eta_c \rho)}{\mathcal{BR}(Z_c \rightarrow J/\psi \pi)}$	$(3.3^{+7.9}_{-1.4}) \times 10^2$	$0.41^{+0.96}_{-0.17}$	$(2.3^{+3.3}_{-1.4}) \times 10^2$	$0.27^{+0.40}_{-0.17}$
$\frac{\mathcal{BR}(Z'_c \rightarrow \eta_c \rho)}{\mathcal{BR}(Z'_c \rightarrow h_c \pi)}$	$(1.2^{+2.8}_{-0.5}) \times 10^2$		$6.6^{+56.8}_{-5.8}$	

$$Z_c(3900) \rightarrow \eta_c \rho$$

Esposito, Guerrieri, AP, PLB 746, 194-201

If molecule

Non-Relativistic Effective Theory, HQET+NRQCD and Hidden gauge Lagrangian
Uncertainty estimated with power counting at NLO



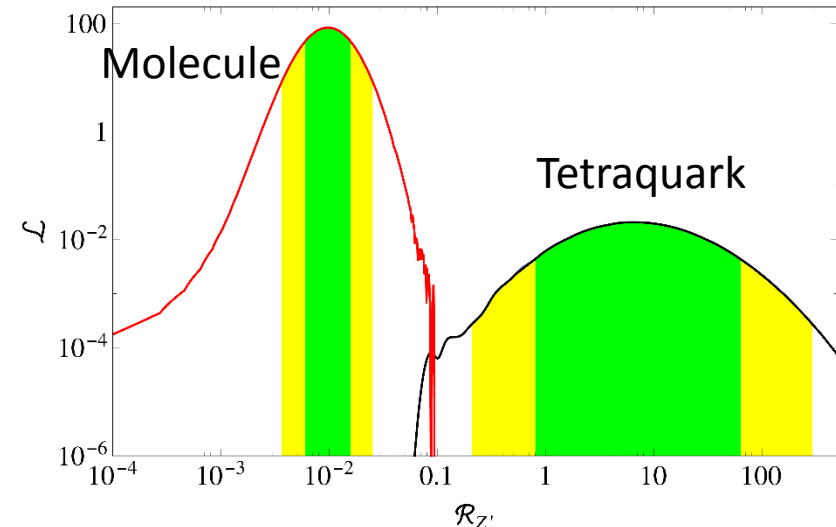
$$\mathcal{L}_{Z_c^{(\prime)}} = \frac{z^{(\prime)}}{2} \langle Z_{\mu,ab}^{(\prime)} \bar{H}_{2b} \gamma^\mu \bar{H}_{1a} \rangle + h.c.,$$

$$\mathcal{L}_{c\bar{c}} = \frac{g_2}{2} \langle \bar{\Psi} H_{1a} \overleftrightarrow{\not{D}} H_{2a} \rangle + \frac{g_1}{2} \langle \bar{\chi}_\mu H_{1a} \gamma^\mu H_{2a} \rangle + h.c.,$$

$$\mathcal{L}_{\rho DD^*} = i\beta \langle H_{1b} v^\mu (\mathcal{V}_\mu - \rho_\mu)_{ba} \bar{H}_{1a} \rangle + i\lambda \langle H_{1b} \sigma^{\mu\nu} F_{\mu\nu}(\rho)_{ba} \bar{H}_{1a} \rangle + h.c.,$$

$$\frac{\mathcal{BR}(Z_c \rightarrow \eta_c \rho)}{\mathcal{BR}(Z_c \rightarrow J/\psi \pi)} = (4.6_{-1.7}^{+2.5}) \times 10^{-2}; \quad \frac{\mathcal{BR}(Z_c' \rightarrow \eta_c \rho)}{\mathcal{BR}(Z_c' \rightarrow h_c \pi)} = (1.0_{-0.4}^{+0.6}) \times 10^{-2}.$$

$$\frac{\mathcal{BR}(Z_c \rightarrow h_c \pi)}{\mathcal{BR}(Z_c' \rightarrow h_c \pi)} = 0.34_{-0.13}^{+0.21}; \quad \frac{\mathcal{BR}(Z_c \rightarrow J/\psi \pi)}{\mathcal{BR}(Z_c' \rightarrow J/\psi \pi)} = 0.35_{-0.21}^{+0.49}$$



Prompt production of $X(3872)$

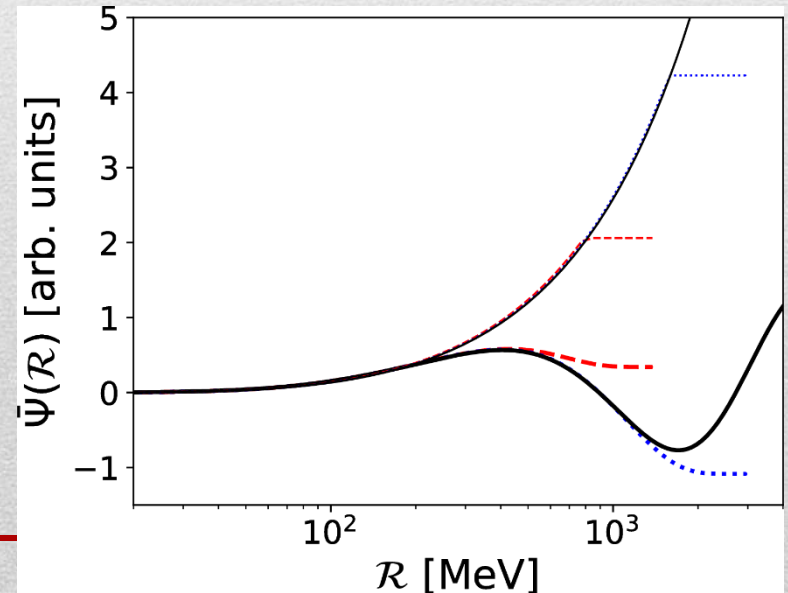
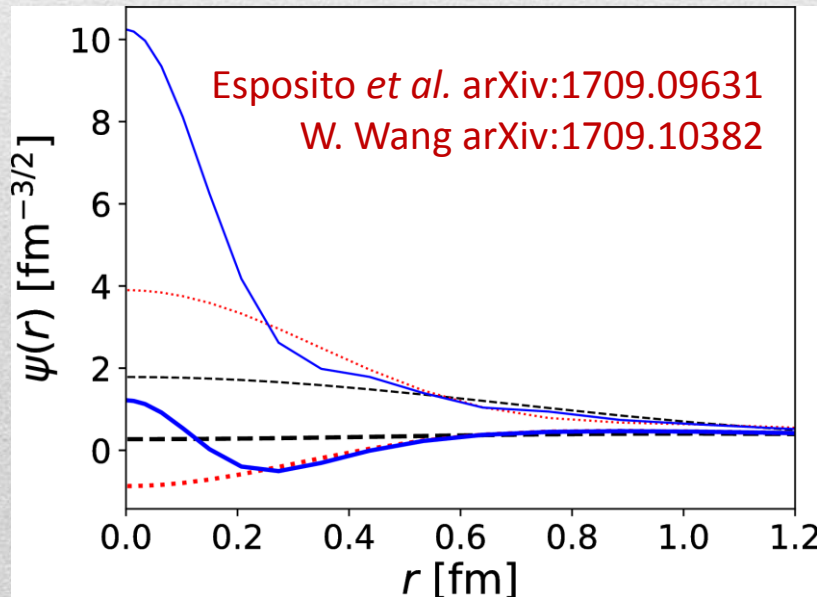
$$\begin{aligned}
 \sigma(\bar{p}p \rightarrow X) &\sim \left| \int d^3\mathbf{k} \langle X | D^0 \bar{D}^{*0}(\mathbf{k}) \rangle \langle D^0 \bar{D}^{*0}(\mathbf{k}) | \bar{p}p \rangle \right|^2 \\
 &\simeq \left| \int_{\mathcal{R}} d^3\mathbf{k} \langle X | D^0 \bar{D}^{*0}(\mathbf{k}) \rangle \langle D^0 \bar{D}^{*0}(\mathbf{k}) | \bar{p}p \rangle \right|^2 \\
 &\leq \int_{\mathcal{R}} d^3\mathbf{k} |\Psi(\mathbf{k})|^2 \int_{\mathcal{R}} d^3\mathbf{k} |\langle D^0 \bar{D}^{*0}(\mathbf{k}) | \bar{p}p \rangle|^2 \\
 &\leq \int_{\mathcal{R}} d^3\mathbf{k} |\langle D^0 \bar{D}^{*0}(\mathbf{k}) | \bar{p}p \rangle|^2
 \end{aligned}$$

The estimate of the k_{max} has been brought back

Albaladejo et al. arXiv:1709.09101

The essence of the argument is that one has to look at the integral of the wave function

$$\int_{\mathcal{R}} d^3\mathbf{k} \psi(\mathbf{k})$$



Towards hybridized tetraquarks

Esposito, AP, Polosa, PLB758, 292

The absence of many of the predicted states might point to the need for **selection rules**

It is unlikely that the **many close-by thresholds** play no role whatsoever

All the well assessed 4-quark resonances lie close and **above** some meson-meson thresholds:

We introduce a **mechanism** that might provide “dynamical selection rules” to explain the presence/absence of resc

	Thr.	δ (MeV)	$A \sqrt{\delta}$ (MeV)	Γ (MeV)
$X(3872)$	$\bar{D}^0 D^{*0}$	0^\dagger	0^\dagger	0^\dagger
$Z_c(3900)$	$\bar{D}^0 D^{*+}$	7.8	27.9	27.9
$Z'_c(4020)$	$\bar{D}^{*0} D^{*+}$	6.7	25.9	24.8 [¶]
$X(4140)$	$J/\psi \phi$	a) 31.6	52.7	28.0
		b) 30.1	54.7	83.0
$Z_b(10610)$	$\bar{B}^0 B^{*+}$	2.7	16.6	18.4
$Z'_b(10650)$	$\bar{B}^{*0} B^{*+}$	1.8	13.4	11.5
$X(5568)$	$B_s^0 \pi^+$	61.4	78.4	21.9
X_{bs}	$B^+ \bar{K}^0$	5.8	24.1	—

We introduce a **mechanism** that might provide “dynamical selection rules” to explain the presence/absence of resonances from the experimental data.

Dictionary – Quark model

L = orbital angular momentum

S = spin $q + \bar{q}$

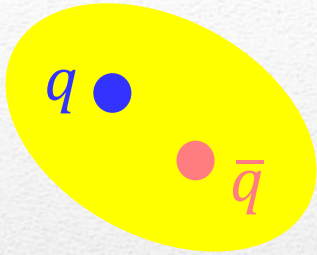
J = total angular momentum
= exp. measured spin

I = isospin = 0 for quarkonia

$$L - S \leq J \leq L + S$$

$$P = (-1)^{L+1}, C = (-1)^{L+S}$$

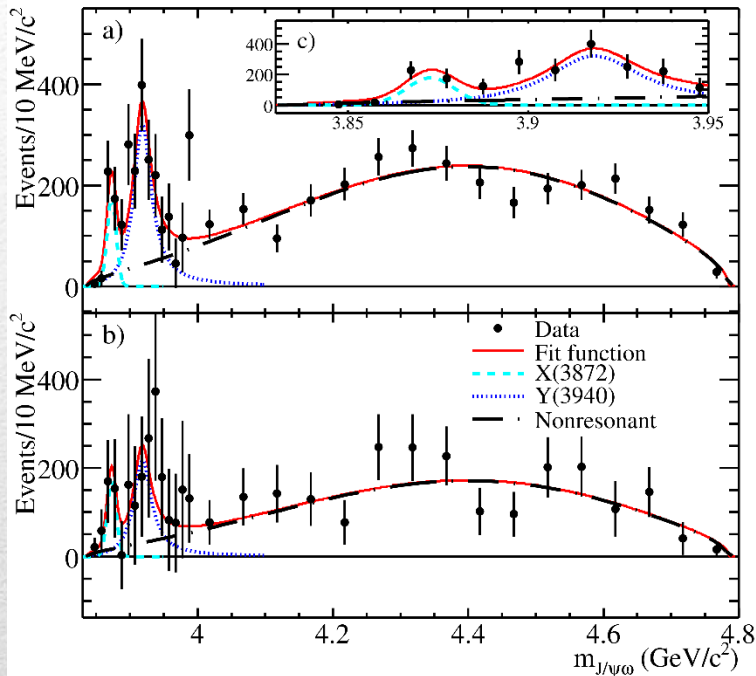
$$G = (-1)^{L+S+I}$$



J^{PC}	L	S	Charmonium ($c\bar{c}$)	Bottomonium ($b\bar{b}$)
0^{-+}	0 (S -wave)	0	$\eta_c(nS)$	$\eta_b(nS)$
1^{--}		1	$\psi(nS)$	$\Upsilon(nS)$
1^{+-}	1 (P -wave)	0	$h_c(nP)$	$h_b(nP)$
0^{++}		1	$\chi_{c0}(nP)$	$\chi_{b0}(nP)$
1^{++}		1	$\chi_{c1}(nP)$	$\chi_{b1}(nP)$
2^{++}		1	$\chi_{c2}(nP)$	$\chi_{b2}(nP)$

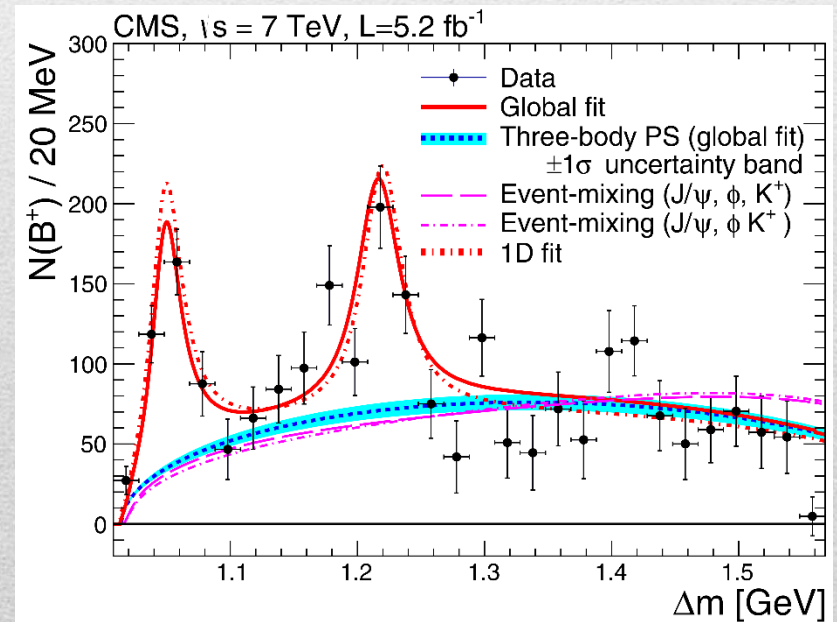
But $J/\psi = \psi(1S)$, $\psi' = \psi(2S)$

Other beasts



One/two peaks seen in $B \rightarrow XK \rightarrow J/\psi \phi K$, close to threshold

$X(3915)$, seen in $B \rightarrow XK \rightarrow J/\psi \omega$
 and $\gamma\gamma \rightarrow X \rightarrow J/\psi \omega$
 $J^{PC} = 0^{++}$, candidate for $\chi_{c0}(2P)$
 But $X(3915) \not\rightarrow D\bar{D}$ as expected,
 and the hyperfine splitting
 $M(2^{++}) - M(0^{++})$ too small



Hybridized tetraquarks

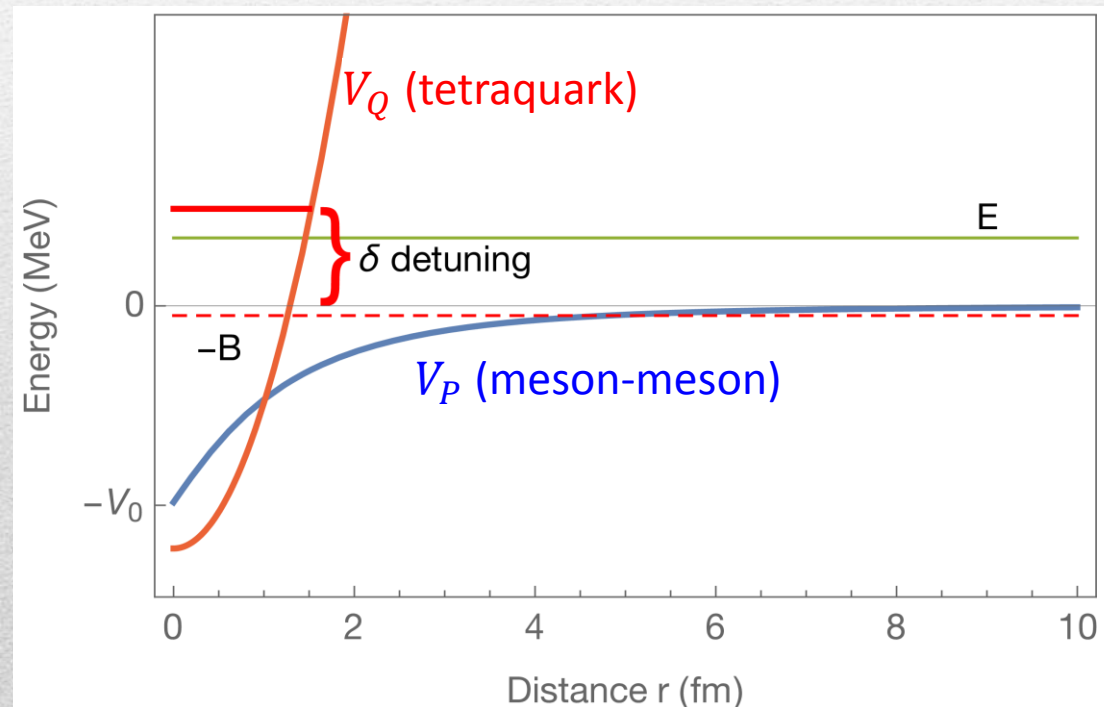
Esposito, AP, Polosa, PLB758, 292

The absence of many of the predicted states might point to the need for **selection rules**

It is unlikely that the **many close-by thresholds** play no role whatsoever

All the well assessed 4-quark resonances lie close and **above** some meson-meson thresholds:

We introduce a **mechanism** that might provide “dynamical selection rules” to explain the presence/absence of resonances from the experimental data



Let P and Q be orthogonal subspaces of the Hilbert space

$$H = H_{PP} + H_{QQ}$$

We have the (weak) scattering length a_P in the open channel.

We add an off-diagonal H_{QP} which connects the two subspaces

Hybridized tetraquarks

Esposito, AP, Polosa, PLB758, 292

$$\Gamma = -16\pi^3 \rho \Im(T) \sim 16\pi^4 \rho |H_{PQ}|^2 \delta \left(\frac{p_1^2}{2M} + \frac{p_2^2}{2M} - \delta \right)$$

The expected width is the average **over momenta that allow for the existence of a tetraquark** $p < \bar{p} = 50 \div 100$ MeV

$$\Gamma \sim A\sqrt{\delta}$$

We therefore expect to see a level if:

- $\delta > 0$ the state **lies above threshold**
- $\delta < \frac{\bar{p}^2}{2M}$, only the **closest threshold** contributes
- The states ψ_Q and ψ_P are **orthogonal**

$X(3872)^+$ falls below threshold, $M(1^{++}) < M(D^{+*}\bar{D}^0)$

$\delta < 0$, so $a > 0 \rightarrow$ **Repulsive interaction**

No charged partners of the $X(3872)$!

Hybridized tetraquarks

Esposito, AP, Polosa, PLB758, 292

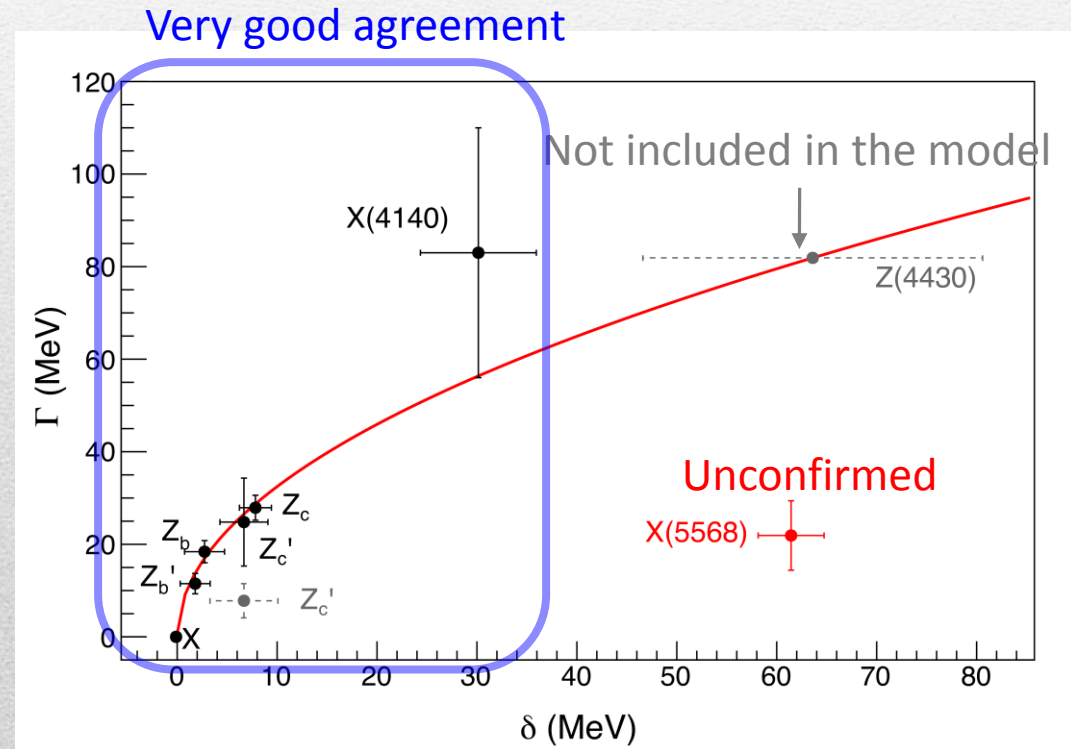
The model works only if no direct transition between closed channel levels can occur
This prevents the straightforward generalization to $L = 1$ and radially excited states
(like the Y s or the $Z(4430)$)

In this picture, a $[bu][\bar{s}\bar{d}]$ state with resonance parameters of the $X(5568)$ observed by D0 is not likely

Also, one has to ensure the orthogonality between the two Hilbert subspaces P and Q .
This might affect the estimate for the $X(4140)$

All the resonances can be fitted with

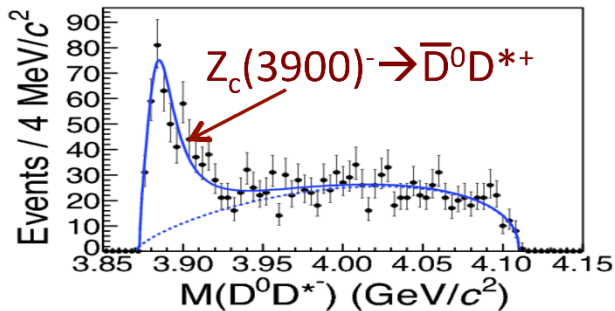
$$A = (10.3 \pm 1.3) \text{ MeV}^{1/2}$$
$$\chi^2/\text{DOF} = 1.2/5$$



$Y(4260) \rightarrow \bar{D}D_1?$

$e^+e^- \rightarrow Y(4260) \rightarrow \pi^- \bar{D}^0 D^{*+}$

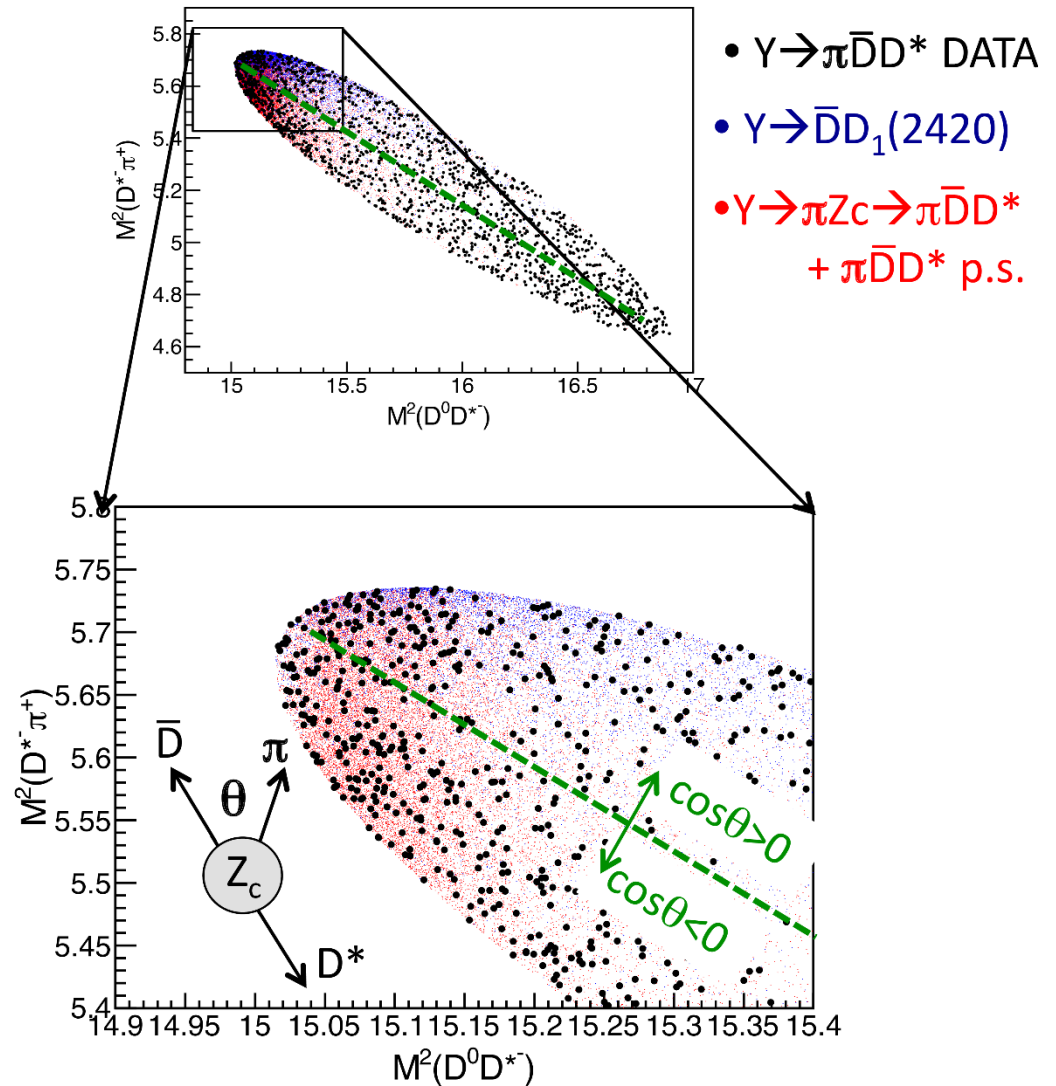
BESIII PRL 112, 022001



$$\mathcal{A} = \frac{N_{|\cos\theta|>0.5} - N_{|\cos\theta|<0.5}}{N_{|\cos\theta|>0.5} + N_{|\cos\theta|<0.5}}$$

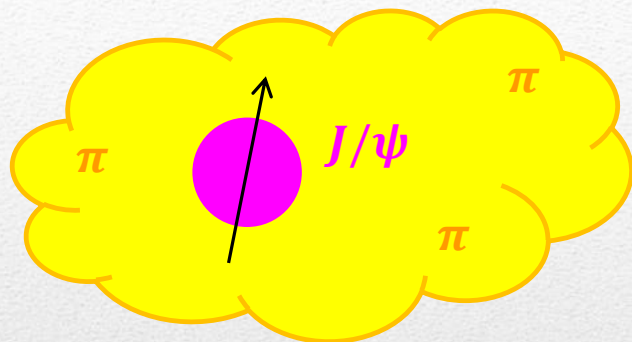
	DD ₁ MC	Z _c +ps MC	data
\mathcal{A}	0.43 ± 0.04	0.02 ± 0.02	0.12 ± 0.06

Not a lot of room for $\bar{D}D_1(2410)$



Hadro-charmonium

Dubynskiy, Voloshin, PLB 666, 344
Dubynskiy, Voloshin, PLB 671, 82
Li, Voloshin, MPLA29, 1450060



Born in the context of QCD multipole expansion

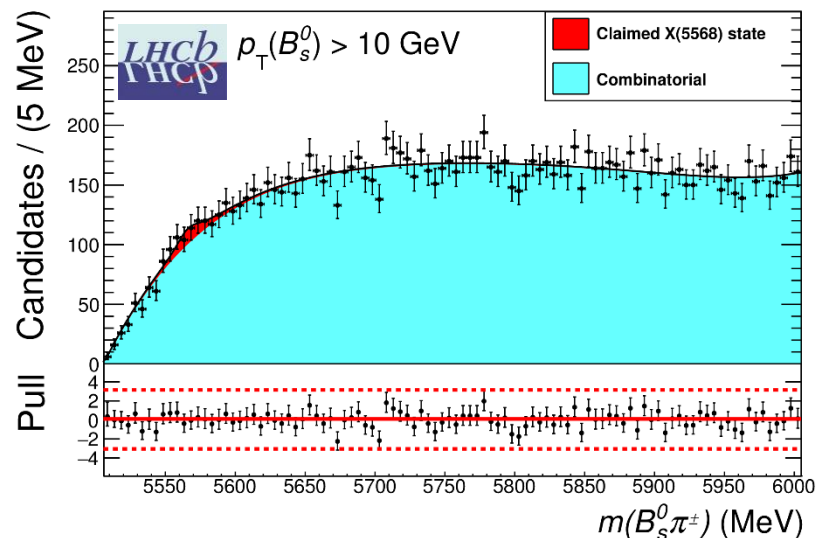
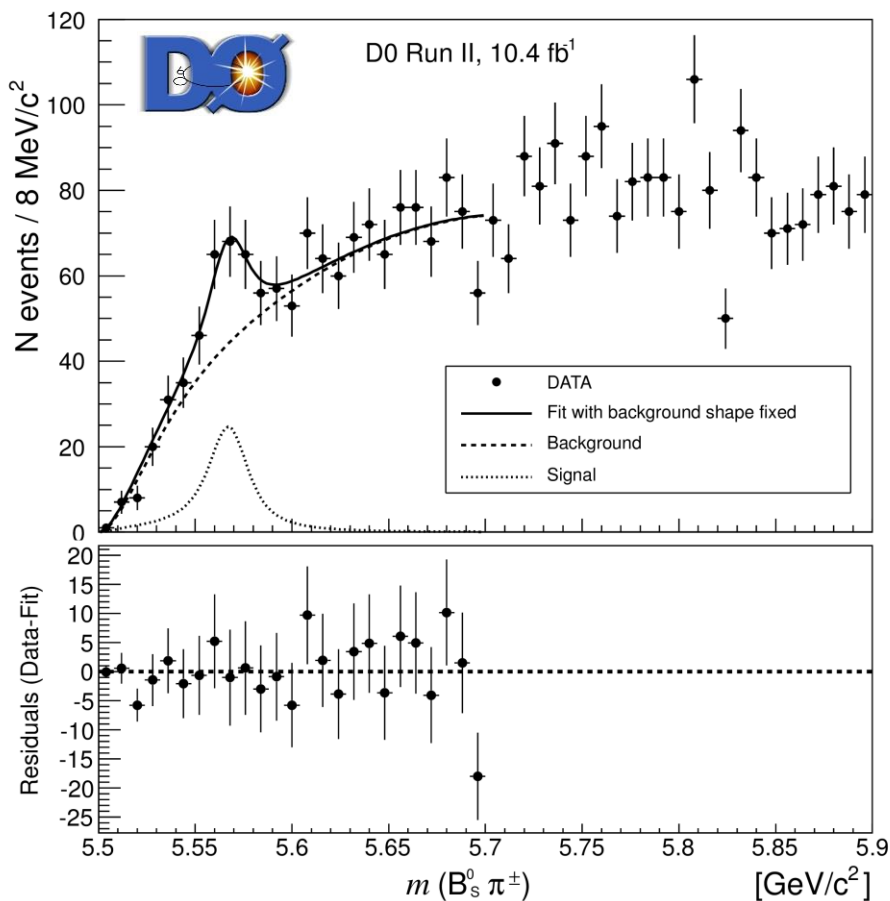
$$H_{eff} = -\frac{1}{2} a_{\psi} E_i^a E_i^a$$
$$a_{\psi} = \langle \psi | (t_c^a - t_{\bar{c}}^a) r_i G r_i (t_c^a - t_{\bar{c}}^a) | \psi \rangle$$

the chromoelectric field interacts with soft light matter (highly excited light hadrons)

A bound state can occur via Van der Waals-like interactions

Expected to decay into core charmonium + light hadrons,
Decay into open charm exponentially suppressed

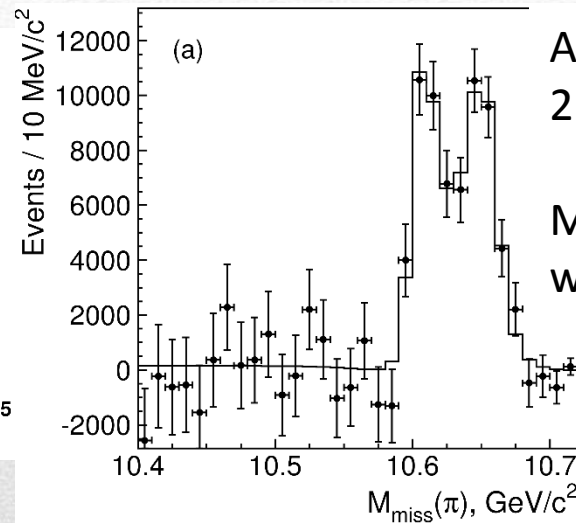
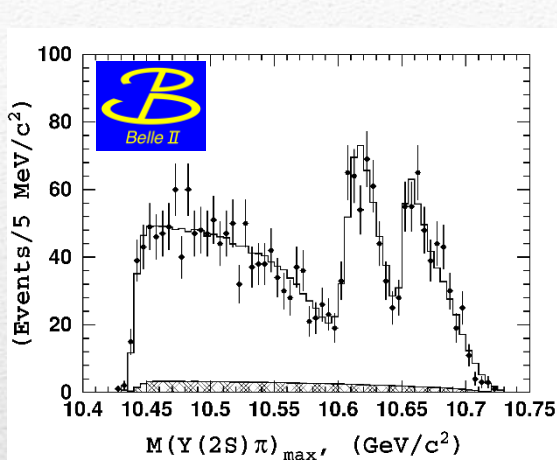
Flavored X(5568)



- A **flavored state** seen in $B_s^0 \pi$ invariant mass by D0 (both $B_s^0 \rightarrow J/\psi \phi$ and $\rightarrow D_s \mu \nu$),
- **not conformed** by LHCb or CMS
- (different kinematics? Compare differential distributions)

Controversy to be solved

Charged Z states: $Z_b(106010)$, $Z'_b(10650)$



Anomalous dipion width in $\Upsilon(5S)$,
2 orders of magnitude larger than $\Upsilon(nS)$

Moreover, observed $\Upsilon(5S) \rightarrow h_b(nP)\pi\pi$
which violates HQSS

2 twin resonances!

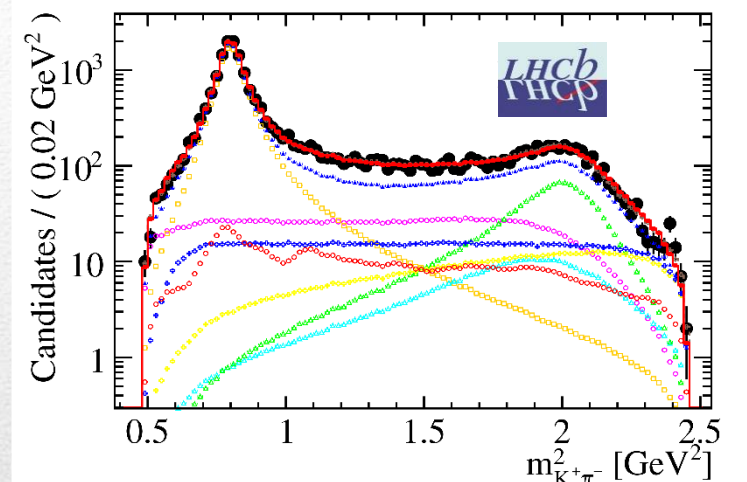
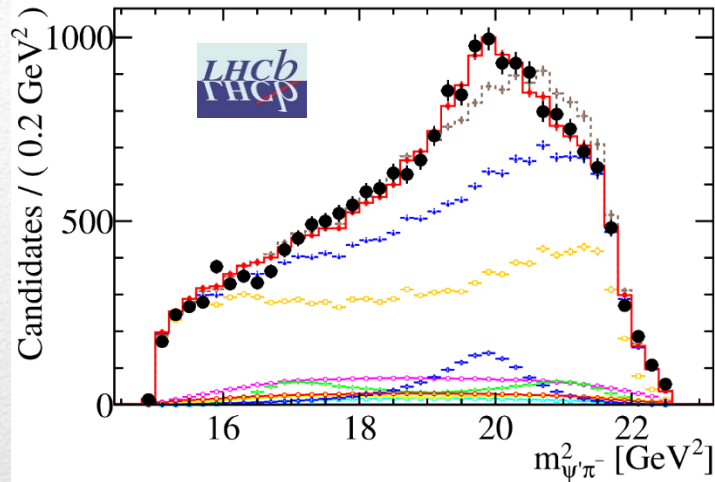
$\Upsilon(5S) \rightarrow Z_b(10610)^+\pi^- \rightarrow \Upsilon(nS)\pi^+\pi^-$, $h_b(nP)\pi^+\pi^-$
and $\rightarrow (BB^*)^+\pi^-$

$M = 10607.2 \pm 2.0$ MeV, $\Gamma = 18.4 \pm 2.4$ MeV

$\Upsilon(5S) \rightarrow Z'_b(10650)^+\pi^- \rightarrow \Upsilon(nS)\pi^+\pi^-$, $h_b(nP)\pi^+\pi^-$
and $\rightarrow \bar{B}^{*0}B^{*+}\pi^-$

$M = 10652.2 \pm 1.5$ MeV, $\Gamma = 11.5 \pm 2.2$ MeV

Charged Z states: Z(4430)



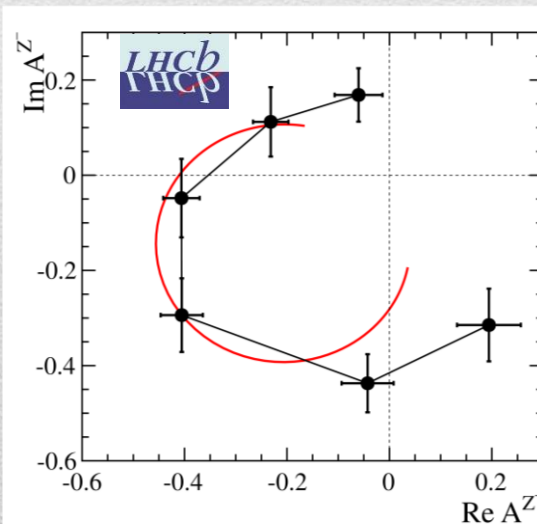
$$Z(4430)^+ \rightarrow \psi(2S) \pi^+$$

$$I^G J^{PC} = 1^+ 1^{+-}$$

$$M = 4475 \pm 7_{-25}^{+15} \text{ MeV}$$

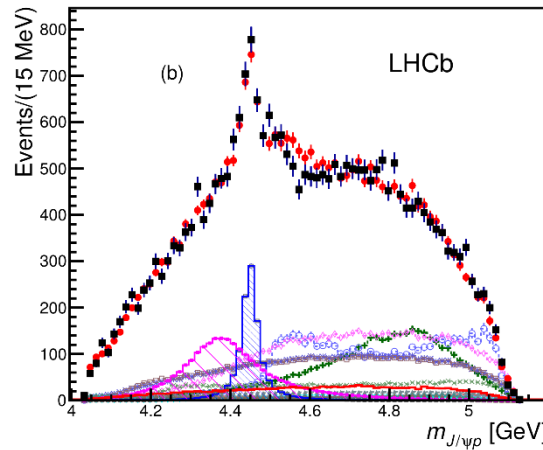
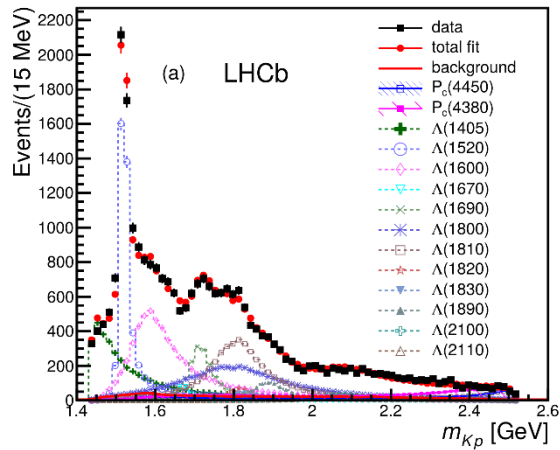
$$\Gamma = 172 \pm 13_{-34}^{+37} \text{ MeV}$$

Far from open charm thresholds



If the amplitude is a free complex number, in each bin of $m_{\psi\pi^-}^2$, the resonant behaviour appears as well

Pentaquarks!



LHCb, PRL 115, 072001
LHCb, PRL 117, 082003

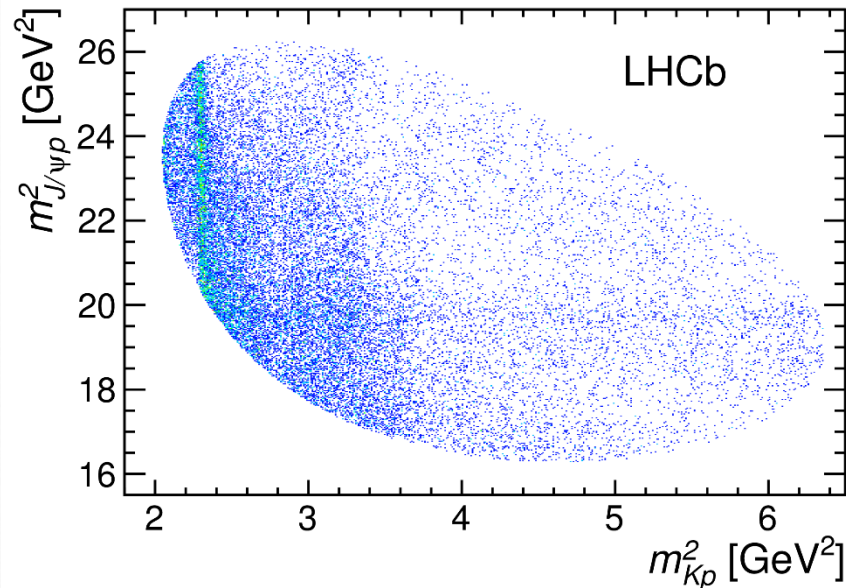
Two states seen in $\Lambda_b \rightarrow (J/\psi p) K^-$,
evidence in $\Lambda_b \rightarrow (J/\psi p) \pi^-$

$$M_1 = 4380 \pm 8 \pm 29 \text{ MeV}$$

$$\Gamma_1 = 205 \pm 18 \pm 86 \text{ MeV}$$

$$M_2 = 4449.8 \pm 1.7 \pm 2.5 \text{ MeV}$$

$$\Gamma_2 = 39 \pm 5 \pm 19 \text{ MeV}$$



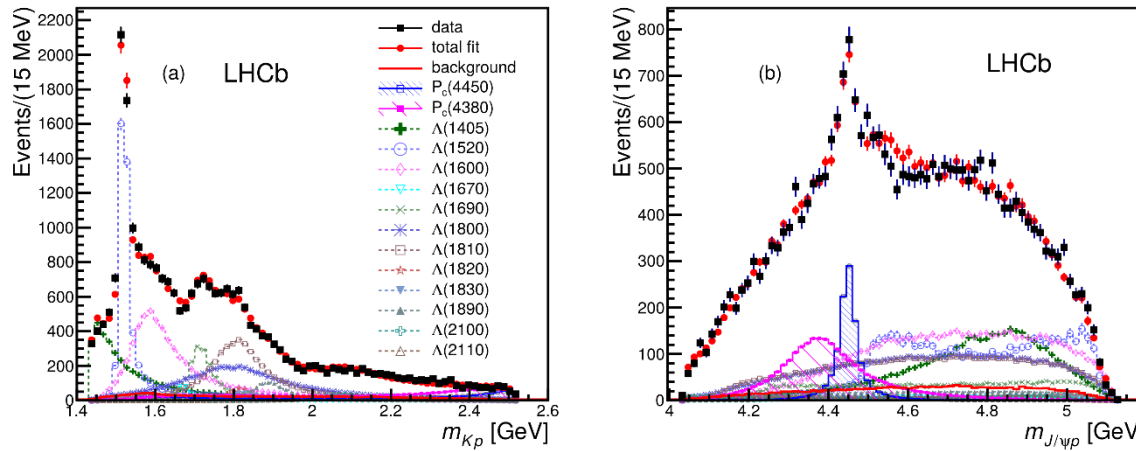
Quantum numbers

$$J^P = \left(\frac{3^-}{2}, \frac{5^+}{2} \right) \text{ or } \left(\frac{3^+}{2}, \frac{5^-}{2} \right) \text{ or } \left(\frac{5^+}{2}, \frac{3^-}{2} \right)$$

Opposite parities needed for the
interference to correctly describe angular
distributions, **low mass region**
contaminated by Λ^* (model dependence?)

No obvious threshold nearby

Pentaquarks!



LHCb, PRL 115, 072001
LHCb, PRL 117, 082003

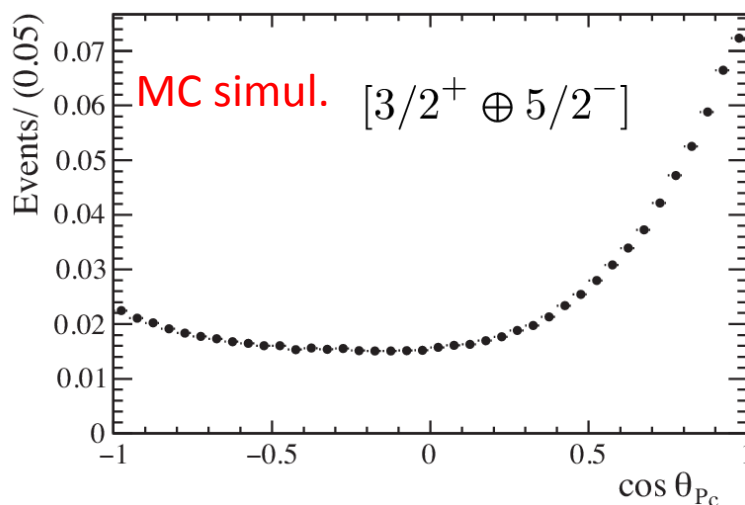
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Opposite parities needed for the interference to correctly describe angular distributions, **low mass region contaminated by Λ^* (model dependence?)**

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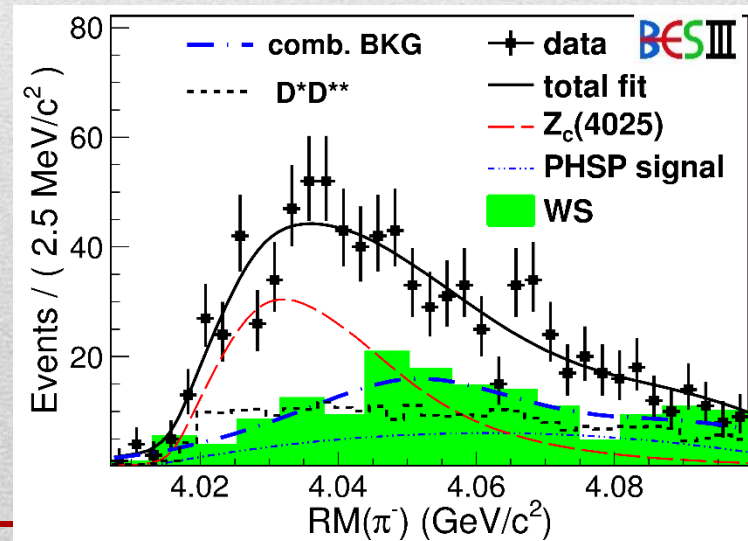
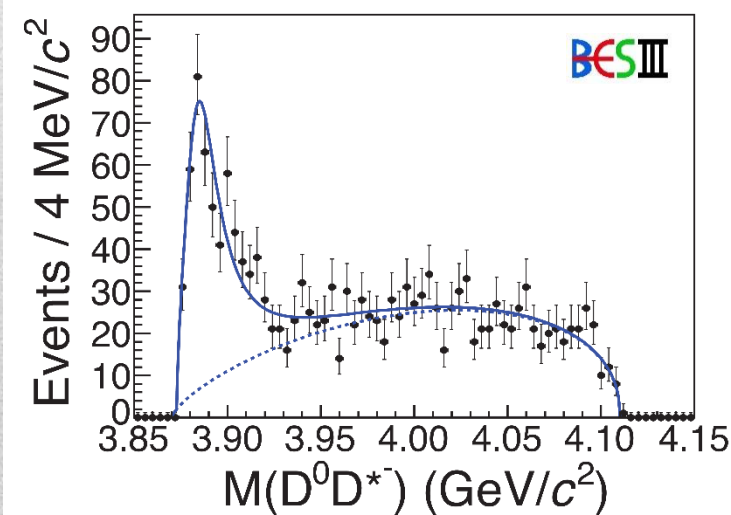
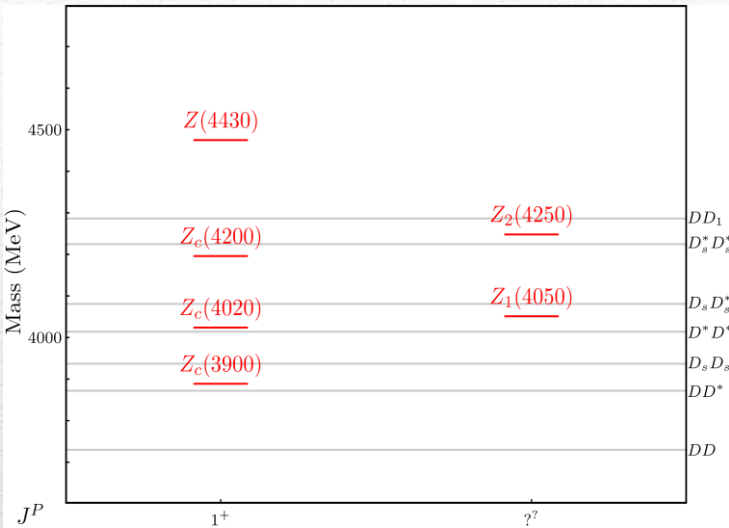
Charged Z states: $Z_c(3900)$, $Z_c'(4020)$

Charged quarkonium-like resonances have been found, **4q needed**

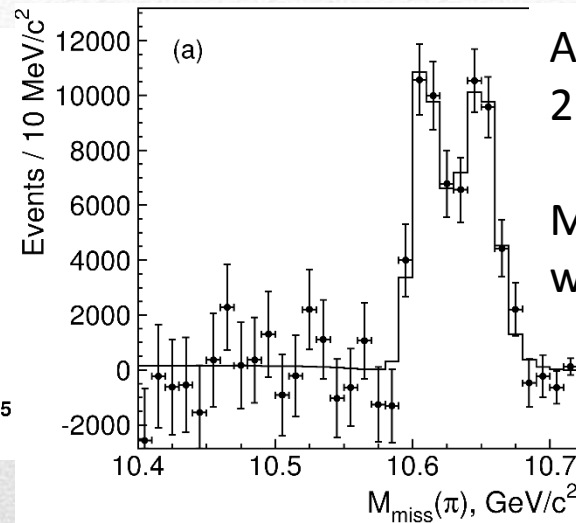
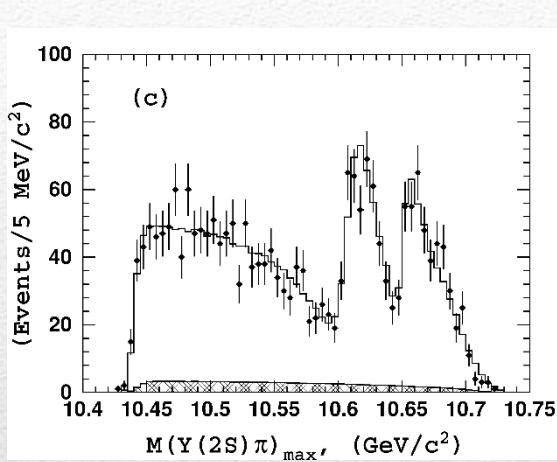
Two states $J^{PC} = 1^{+-}$ appear
slightly above $D^{(*)}D^*$ thresholds

$e^+e^- \rightarrow Z_c(3900)^+\pi^- \rightarrow J/\psi \pi^+\pi^-$ and $\rightarrow (DD^*)^+\pi^-$
 $M = 3888.7 \pm 3.4 \text{ MeV}, \Gamma = 35 \pm 7 \text{ MeV}$

$e^+e^- \rightarrow Z_c'(4020)^+\pi^- \rightarrow h_c \pi^+\pi^-$ and $\rightarrow \bar{D}^{*0}D^{*+}\pi^-$
 $M = 4023.9 \pm 2.4 \text{ MeV}, \Gamma = 10 \pm 6 \text{ MeV}$



Charged Z states: $Z_b(106010)$, $Z'_b(10650)$



Anomalous dipion width in $\Upsilon(5S)$,
2 orders of magnitude larger than $\Upsilon(nS)$

Moreover, observed $\Upsilon(5S) \rightarrow h_b(nP)\pi\pi$
which violates HQSS

2 twin resonances!

$$\Upsilon(5S) \rightarrow Z_b(10610)^+\pi^- \rightarrow \Upsilon(nS)\pi^+\pi^-, h_b(nP)\pi^+\pi^-$$

$$\text{and } \rightarrow (BB^*)^+\pi^-$$

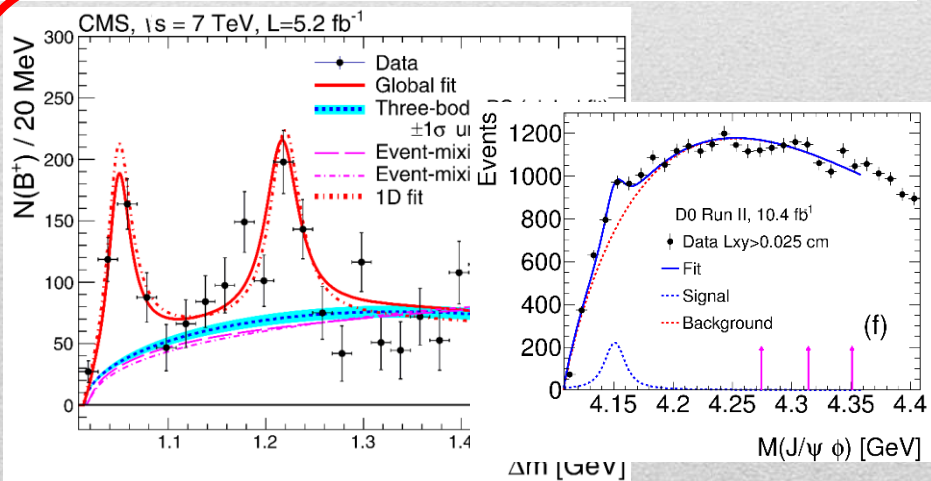
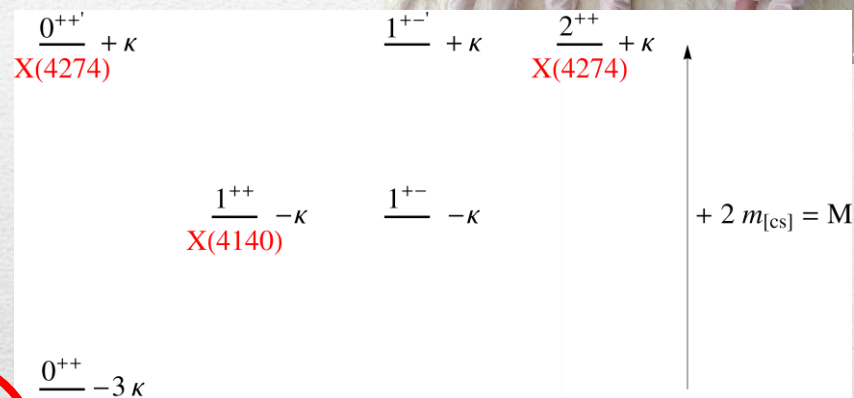
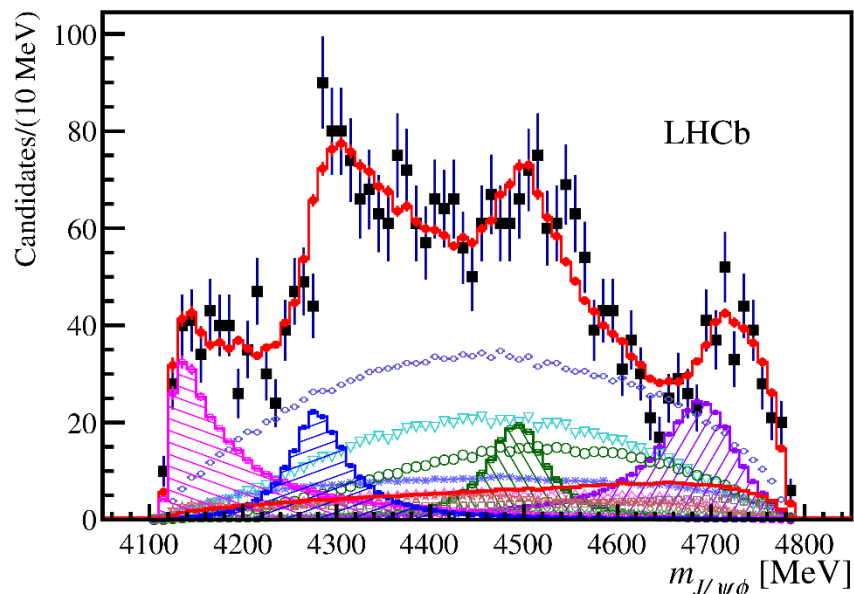
$$M = 10607.2 \pm 2.0 \text{ MeV}, \Gamma = 18.4 \pm 2.4 \text{ MeV}$$

$$\Upsilon(5S) \rightarrow Z'_b(10650)^+\pi^- \rightarrow \Upsilon(nS)\pi^+\pi^-, h_b(nP)\pi^+\pi^-$$

$$\text{and } \rightarrow \bar{B}^{*0}B^{*+}\pi^-$$

$$M = 10652.2 \pm 1.5 \text{ MeV}, \Gamma = 11.5 \pm 2.2 \text{ MeV}$$

Tetraquark: the $c\bar{c}s\bar{s}$ states



Good description of the spectrum **but** one has to assume the axial assignment for the $X(4274)$ to be incorrect (two unresolved states with 0^{++} and 2^{++})

Maiani, Polosa and Riquer, PRD 94, 054026

Much narrower than LHCb! Look for prompt!

State	M (MeV)	Γ (MeV)	J^{PC}	Process (mode)	Experiment ($\#\sigma$)
$X(3823)$	3823.1 ± 1.9	< 24	$?^{? -}$	$B \rightarrow K(\chi_{c1}\gamma)$	Belle ^[23] (4.0)
$X(3872)$	3871.68 ± 0.17	< 1.2	1^{++}	$B \rightarrow K(\pi^+\pi^-J/\psi)$	Belle ^[24,25] (>10), BABAR ^[26] (8.6)
				$p\bar{p} \rightarrow (\pi^+\pi^-J/\psi) \dots$	CDF ^[27,28] (11.6), D0 ^[29] (5.2)
				$pp \rightarrow (\pi^+\pi^-J/\psi) \dots$	LHCb ^[30,31] (np)
				$B \rightarrow K(\pi^+\pi^-\pi^0J/\psi)$	Belle ^[32] (4.3), BABAR ^[33] (4.0)
				$B \rightarrow K(\gamma J/\psi)$	Belle ^[34] (5.5), BABAR ^[35] (3.5)
					LHCb ^[36] (>10)
				$B \rightarrow K(\gamma\psi(2S))$	BABAR ^[35] (3.6), Belle ^[34] (0.2)
					LHCb ^[36] (4.4)
				$B \rightarrow K(D\bar{D}^*)$	Belle ^[37] (6.4), BABAR ^[38] (4.9)
$Z_c(3900)^+$	3888.7 ± 3.4	35 ± 7	1^{+-}	$Y(4260) \rightarrow \pi^-(D\bar{D}^*)^+$	BES III ^[39] (np)
				$Y(4260) \rightarrow \pi^-(\pi^+J/\psi)$	BES III ^[40] (8), Belle ^[41] (5.2)
					CLEO data ^[42] (>5)
$Z_c(4020)^+$	4023.9 ± 2.4	10 ± 6	1^{+-}	$Y(4260) \rightarrow \pi^-(\pi^+h_c)$	BES III ^[43] (8.9)
				$Y(4260) \rightarrow \pi^-(D^*\bar{D}^*)^+$	BES III ^[44] (10)
$Y(3915)$	3918.4 ± 1.9	20 ± 5	0^{++}	$B \rightarrow K(\omega J/\psi)$	Belle ^[45] (8), BABAR ^[33,46] (19)
				$e^+e^- \rightarrow e^+e^-(\omega J/\psi)$	Belle ^[47] (7.7), BABAR ^[48] (7.6)
$Z(3930)$	3927.2 ± 2.6	24 ± 6	2^{++}	$e^+e^- \rightarrow e^+e^-(D\bar{D})$	Belle ^[49] (5.3), BABAR ^[50] (5.8)
$X(3940)$	3942_{-8}^{+9}	37_{-17}^{+27}	$?^{?+}$	$e^+e^- \rightarrow J/\psi(D\bar{D}^*)$	Belle ^[51,52] (6)
$Y(4008)$	3891 ± 42	255 ± 42	1^{--}	$e^+e^- \rightarrow (\pi^+\pi^-J/\psi)$	Belle ^[41,53] (7.4)
$Z(4050)^+$	4051_{-43}^{+24}	82_{-55}^{+51}	$?^{?+}$	$\bar{B}^0 \rightarrow K^-(\pi^+\chi_{c1})$	Belle ^[54] (5.0), BABAR ^[55] (1.1)
$Y(4140)$	4145.6 ± 3.6	14.3 ± 5.9	$?^{?+}$	$B^+ \rightarrow K^+(\phi J/\psi)$	CDF ^[56,57] (5.0), Belle ^[58] (1.9), LHCb ^[59] (1.4), CMS ^[60] (>5) D0 ^[61] (3.1)
$X(4160)$	4156_{-25}^{+29}	139_{-65}^{+113}	$?^{?+}$	$e^+e^- \rightarrow J/\psi(D^*\bar{D}^*)$	Belle ^[52] (5.5)
$Z(4200)^+$	4196_{-30}^{+35}	370_{-110}^{+99}	1^{+-}	$\bar{B}^0 \rightarrow K^-(\pi^+J/\psi)$	Belle ^[62] (7.2)

State	M (MeV)	Γ (MeV)	J^{PC}	Process (mode)	Experiment ($\#\sigma$)
$Y(4220)$	4196_{-30}^{+35}	39 ± 32	1^{--}	$e^+e^- \rightarrow (\pi^+\pi^-h_c)$	BES III data ^[63,64] (4.5)
$Y(4230)$	4230 ± 8	38 ± 12	1^{--}	$e^+e^- \rightarrow (\chi_{c0}\omega)$	BES III ^[65] (>9)
$Z(4250)^+$	4248_{-45}^{+185}	177_{-72}^{+321}	$?^{?+}$	$\bar{B}^0 \rightarrow K^-(\pi^+\chi_{c1})$	Belle ^[54] (5.0), BABAR ^[55] (2.0)
$Y(4260)$	4250 ± 9	108 ± 12	1^{--}	$e^+e^- \rightarrow (\pi\pi J/\psi)$	BABAR ^[66,67] (8), CLEC ^[68,69] (11) Belle ^[41,53] (15), BES III ^[40] (np)
				$e^+e^- \rightarrow (f_0(980)J/\psi)$	BABAR ^[67] (np), Belle ^[41] (np)
				$e^+e^- \rightarrow (\pi^-Z_c(3900)^+)$	BES III ^[40] (8), Belle ^[41] (5.2)
				$e^+e^- \rightarrow (\gamma X(3872))$	BES II ^[70] (5.3)
$Y(4290)$	4293 ± 9	222 ± 67	1^{--}	$e^+e^- \rightarrow (\pi^+\pi^-h_c)$	BES III data ^[63,64] (np)
$X(4350)$	$4350.6_{-5.1}^{+4.6}$	13_{-10}^{+18}	$0/2^{?+}$	$e^+e^- \rightarrow e^+e^-(\phi J/\psi)$	Belle ^[58] (3.2)
$Y(4360)$	4354 ± 11	78 ± 16	1^{--}	$e^+e^- \rightarrow (\pi^+\pi^- \psi(2S))$	Belle ^[71] (8), BABAR ^[72] (np)
$Z(4430)^+$	4478 ± 17	180 ± 31	1^{+-}	$\bar{B}^0 \rightarrow K^-(\pi^+\psi(2S))$	Belle ^[73,74] (6.4), BABAR ^[75] (2.4) LHCb ^[76] (13.9)
				$\bar{B}^0 \rightarrow K^-(\pi^+J/\psi)$	Belle ^[62] (4.0)
$Y(4630)$	4634_{-11}^{+9}	92_{-32}^{+41}	1^{--}	$e^+e^- \rightarrow (\Lambda_c^+\bar{\Lambda}_c^-)$	Belle ^[77] (8.2)
$Y(4660)$	4665 ± 10	53 ± 14	1^{--}	$e^+e^- \rightarrow (\pi^+\pi^- \psi(2S))$	Belle ^[71] (5.8), BABAR ^[72] (5)
$Z_b(10610)^+$	10607.2 ± 2.0	18.4 ± 2.4	1^{+-}	$\Upsilon(5S) \rightarrow \pi(\pi\Upsilon(nS))$	Belle ^[78,79] (>10)
				$\Upsilon(5S) \rightarrow \pi^-(\pi^+h_b(nP))$	Belle ^[78] (16)
				$\Upsilon(5S) \rightarrow \pi^-(B\bar{B}^*)^+$	Belle ^[80] (8)
$Z_b(10650)^+$	10652.2 ± 1.5	11.5 ± 2.2	1^{+-}	$\Upsilon(5S) \rightarrow \pi^-(\pi^+\Upsilon(nS))$	Belle ^[78] (>10)
				$\Upsilon(5S) \rightarrow \pi^-(\pi^+h_b(nP))$	Belle ^[78] (16)
				$\Upsilon(5S) \rightarrow \pi^-(B^*\bar{B}^*)^+$	Belle ^[80] (6.8)

**Guerrieri, AP, Piccinini, Polosa,
IJMPA 30, 1530002**

Interactive tools

- Completed projects are fully documented on interactive portals
- These include description on physics, conventions, formalism, etc.
- The web pages contain source codes with detailed explanation how to use them. Users can run codes online, change parameters, display results.

<http://www.indiana.edu/~jpac/>

Joint Physics Analysis Center

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This project is supported by NSF

$$\pi N \rightarrow \pi N$$

Formalism

The pion-nucleon scattering is a function of 2 variables. The first is the beam momentum in the laboratory frame p_{lab} (in GeV) or the total energy squared $s = W^2$ (in GeV^2). The second is the cosine of



Resources

- Publications: [Mat15a] and [Wor12a]
- SAID partial waves: compressed zip file
- C/C++: C/C++ file
- Input file: param.txt
- Output files: output0.txt, output1.txt, SigTot.txt, Observables0.txt, Observables1.txt
- Contact person: Vincent Mathieu
- Last update: June 2016

The SAID partial waves are in the format provided online on the SAID webpage :

p_{lab} δ $\epsilon(\delta)$ $1 - \eta^2$ $\epsilon(1 - \eta^2)$ Re PW Im PW SGT SGR

δ and η are the phase-shift and the inelasticity. $\epsilon(x)$ is the error on x . SGT is the total cross section and SGR is the total reaction cross section.

Format of the input and output files: [show/hide]
Description of the C/C++ code: [show/hide]

Simulation

Range of the running variable:

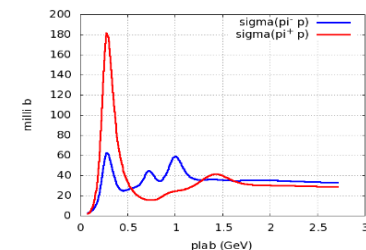
s in GeV^2 (min max step)	1,2	:	6	:	0,01
p_{lab} in GeV (min max step)	0,1	:	4	:	0,01
ν in GeV (min max step)	0,3	:	4	:	0,01
t in GeV^2 (min max step)	-1	:	0	:	0,01

The fixed variable:

t in GeV^2

p_{lab} in GeV

Results



Lineshapes at 4230

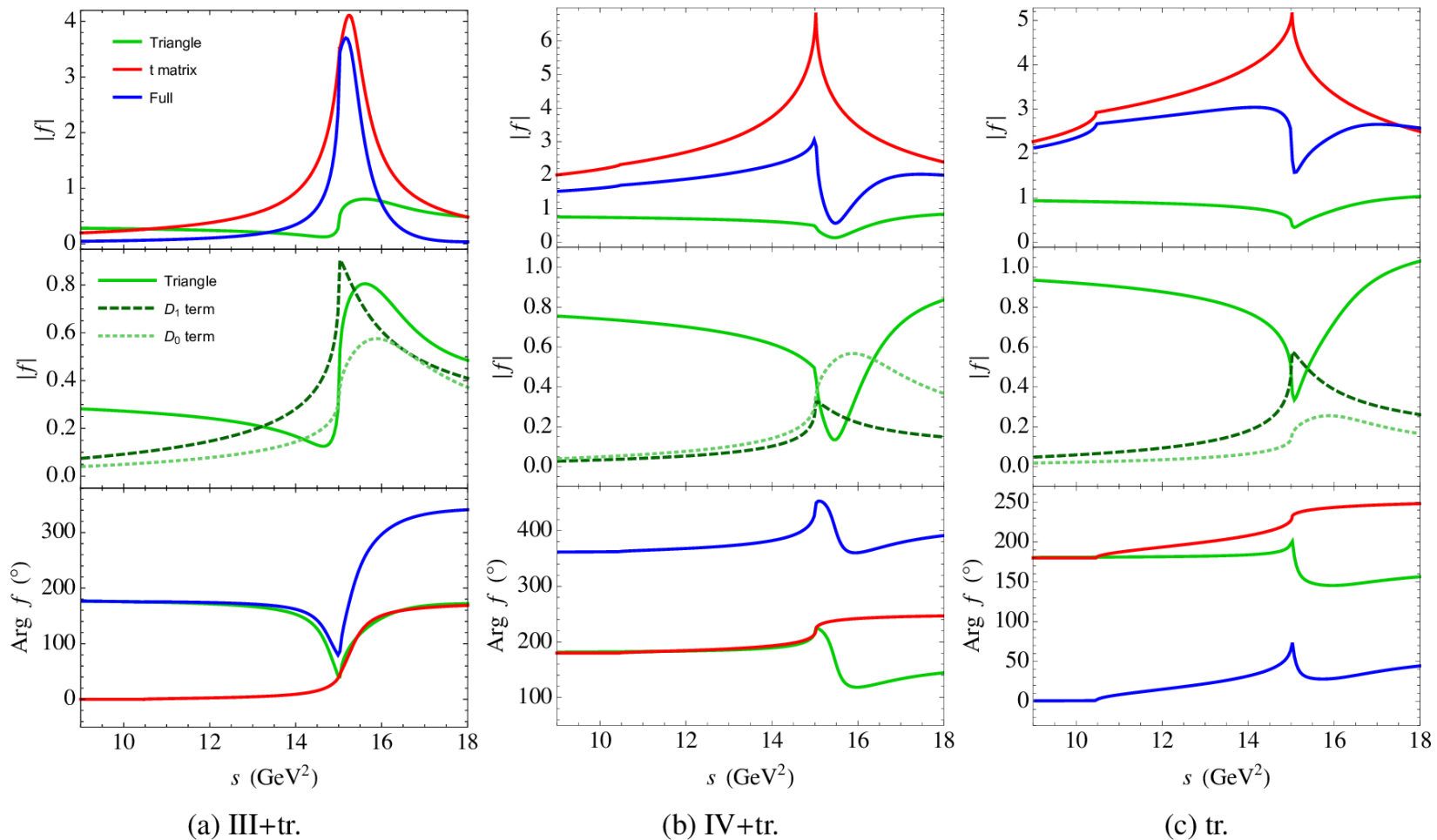


Figure 8: Same as Figure 7, but for $E_{CM} = 4.23$ GeV.

Counting rules

Brodsky, Lebed PRD91, 114025

- Exotic states can be produced in threshold regions in e^+e^- , electroproduction, hadronic beam facilities and are best characterized by cross section ratios
- Two examples:

$$1) \frac{\sigma(e^+e^- \rightarrow Z_c^+ \pi^-)}{\sigma(e^+e^- \rightarrow \mu^+ \mu^-)} \propto \frac{1}{s^6} \text{ as } s \rightarrow \infty$$

$$2) \frac{\sigma(e^+e^- \rightarrow Z_c^+(\bar{c}c\bar{d}u) + \pi^-(\bar{u}d))}{\sigma(e^+e^- \rightarrow \Lambda_c(cud) + \bar{\Lambda}_c(\bar{c}\bar{u}\bar{d}))} \rightarrow \text{const as } s \rightarrow \infty$$

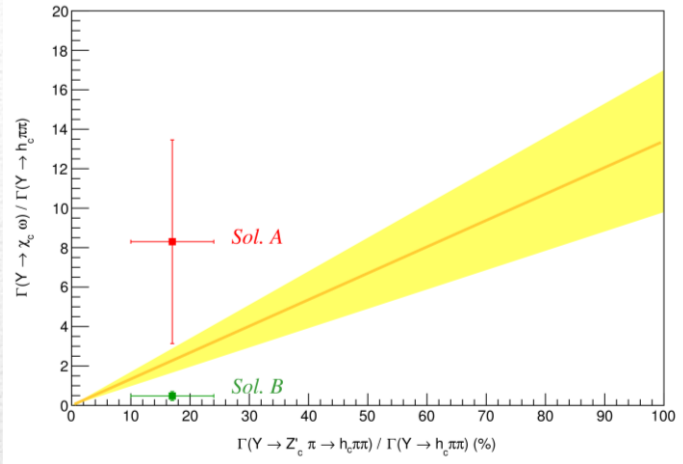
- Ratio numerically smaller if Z_c behaves like weakly-bound dimeson molecule instead of diquark-antidiquark bound state due to weaker meson color van der Waals forces

Different estimates close to thresholds, and in presence of annihilating $q \bar{q}$

Guo, Meissner, Wang, Yang, 1607.04020

Voloshin PRD94, 074042

Tetraquark: the $Y(4220)$



$$\langle \chi_{c0}(p) \omega(\eta, q) | Y(\lambda, P) \rangle = g_\chi \eta \cdot \lambda,$$

$$\langle Z'_c(\eta, q) \pi(p) | Y(\lambda, P) \rangle = g_Z \eta \cdot \lambda \frac{P \cdot p}{f_\pi M_Y},$$

$$\langle h_c(\eta, q) \sigma(p) | Y(\lambda, P) \rangle = g_h \varepsilon_{\mu\nu\rho\sigma} \eta^\mu \lambda^\nu \frac{P^\rho q^\sigma}{P \cdot q},$$

$$\langle \pi(q) \pi(p) | \sigma(P) \rangle = \frac{P^2}{2f_\pi},$$

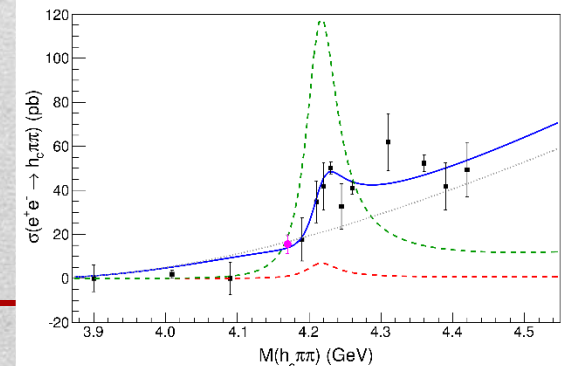
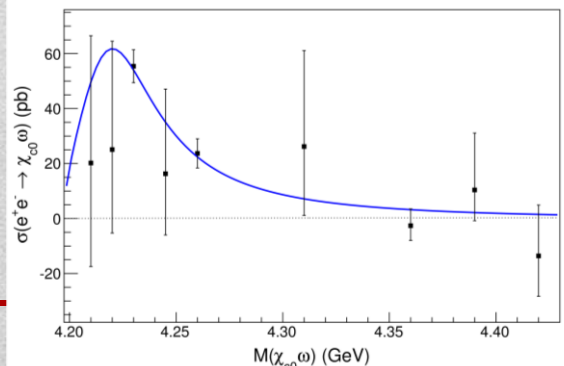
$$\frac{\Gamma(Y(4220) \rightarrow \chi_{c0} \omega)}{\Gamma(Y(4220) \rightarrow h_c \pi^+ \pi^-)} = (13.4 \pm 3.6) \times R_{YZ} = 2.3 \pm 1.2.$$

$$\frac{\Gamma(Y(4220) \rightarrow Z'_c{}^\pm \pi^\mp \rightarrow h_c \pi^+ \pi^-)}{\Gamma(Y(4220) \rightarrow h_c \sigma \rightarrow h_c \pi^+ \pi^-)} = 4.8 \pm 3.5,$$

A state apparently breaking HQSS has been observed

Compatible to be the Y_3 state

Faccini, Filaci, Guerrieri, AP, Polosa, PRD 91, 117501



Tetraquark: the b sector

Ali, Maiani, Piccinini, Polosa, Riquer PRD91 017502

$$\begin{aligned}M(Z'_b) - M(Z_b) &= 2\kappa_b \\M(Z'_c) - M(Z_c) &= 2\kappa_c \sim 120 \text{ MeV} \\ \kappa_b : \kappa_c &= M_c : M_b \sim 0.30\end{aligned}$$

$$2\kappa_b \sim 36 \text{ MeV, vs. } 45 \text{ MeV (exp.)}$$

$$\begin{aligned}Z_b &= \frac{\alpha |1_{q\bar{q}}0_{b\bar{b}}\rangle - \beta |0_{q\bar{q}}1_{b\bar{b}}\rangle}{\sqrt{2}} \\ Z'_b &= \frac{\alpha |1_{q\bar{q}}0_{b\bar{b}}\rangle + \beta |0_{q\bar{q}}1_{b\bar{b}}\rangle}{\sqrt{2}}\end{aligned}$$

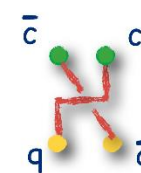
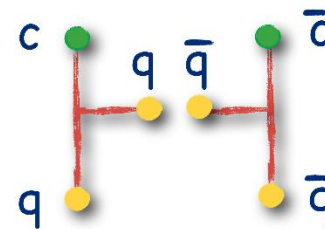
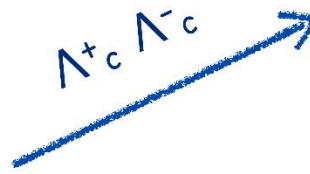
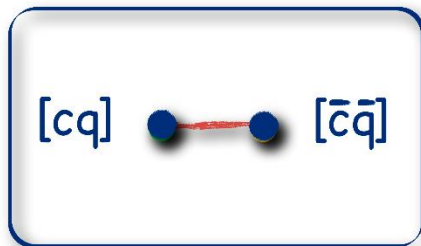
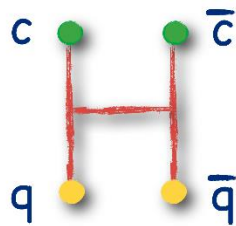
Data on $\Upsilon(5S) \rightarrow \Upsilon(nS)\pi\pi$ and $\Upsilon(5S) \rightarrow h_b(nP)\pi\pi$ strongly favor $\alpha = \beta$

Baryonium

C. Sabelli

a structure $[cq][\bar{c}\bar{q}]$ can explain the dominance of baryon channel

Isospin violation expected,
 $\alpha_s(m_c) \ll 1$



Rossi, Veneziano,
 NPB 123, 507;
 Phys.Rept. 63, 149;
 PLB70, 255

$$\frac{B(Y(4660) \rightarrow \Lambda_c^+ \Lambda_c^-)}{B(Y(4660) \rightarrow \psi(2S)\pi\pi)} = 25 \pm 7$$

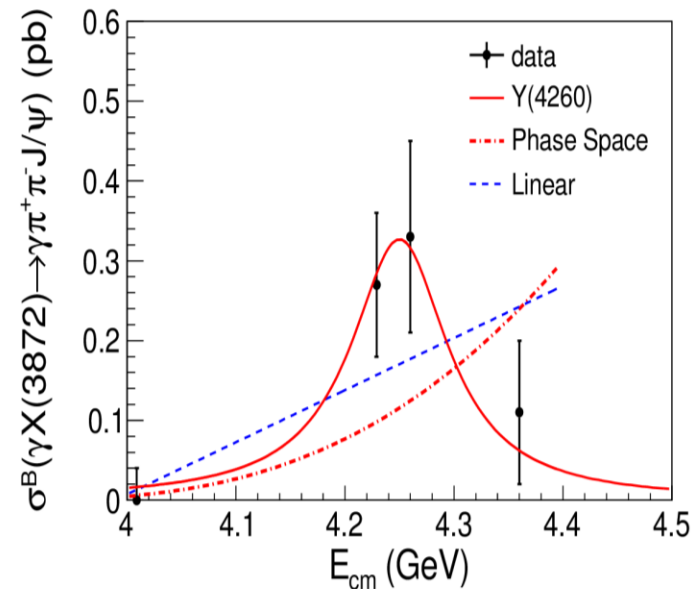
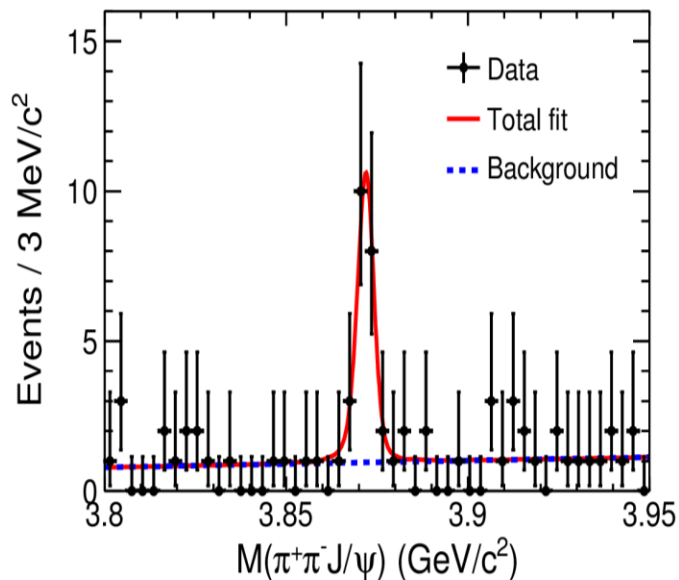
Cotugno, Faccini, Polosa, Sabelli,
 PRL 104, 132005

$Y(4260) \rightarrow \gamma X(3872)$

M. Ablikim et al., Phys. Rev. Lett. 112 (2014) 092001

F. Piccinini

BESIII: $e^+e^- \rightarrow Y(4260) \rightarrow X(3872)\gamma$



With $\mathcal{B}[X(3872) \rightarrow \pi^+\pi^- J/\psi] = 5\%$

$$\frac{\mathcal{B}[Y(4260) \rightarrow \gamma X(3872)]}{\mathcal{B}[Y(4260) \rightarrow \pi^+\pi^- J/\psi]} = 0.1$$

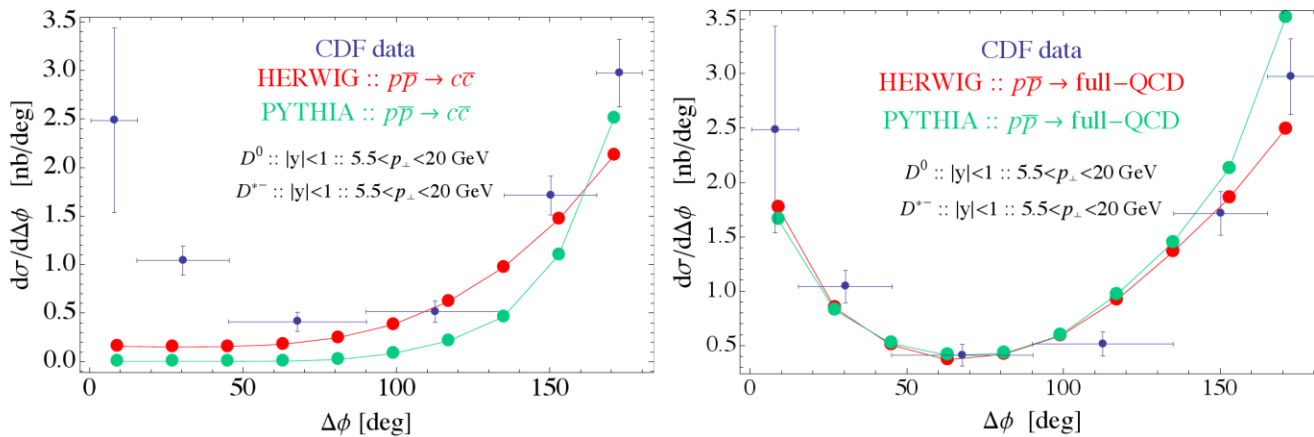
Strong indication that $Y(4260)$ and $X(3872)$ share a similar structure

Tuning of MC

Monte Carlo simulations

A. Esposito

- We compare the $D^0 D^{*-}$ pairs produced as a function of relative azimuthal angle with the results from CDF:



The $c\bar{c}$ run underestimate the low angles (low- k_\perp) region!

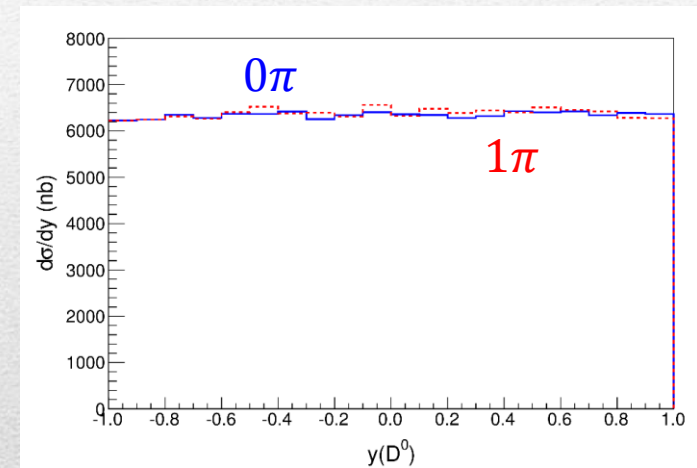
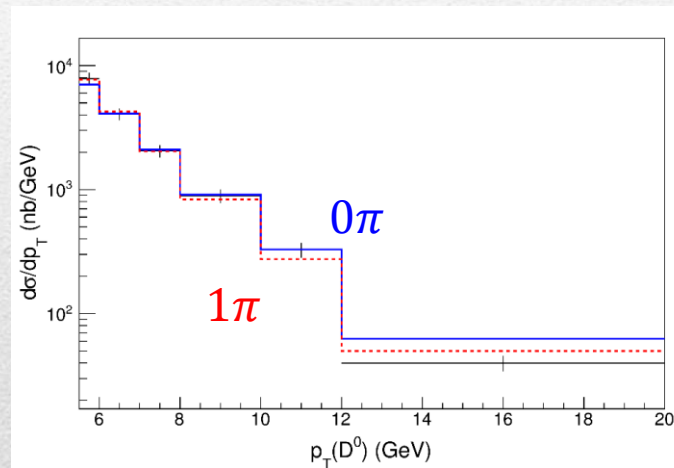
Such distributions of charm mesons are available at Tevatron
No distribution has been published (yet) at LHC

Tuning pions

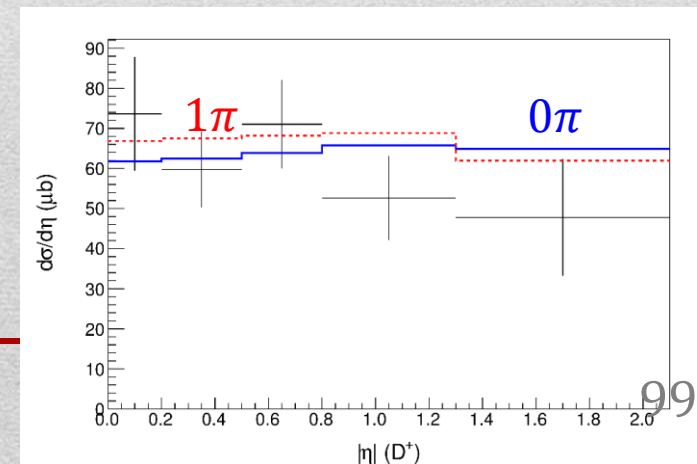
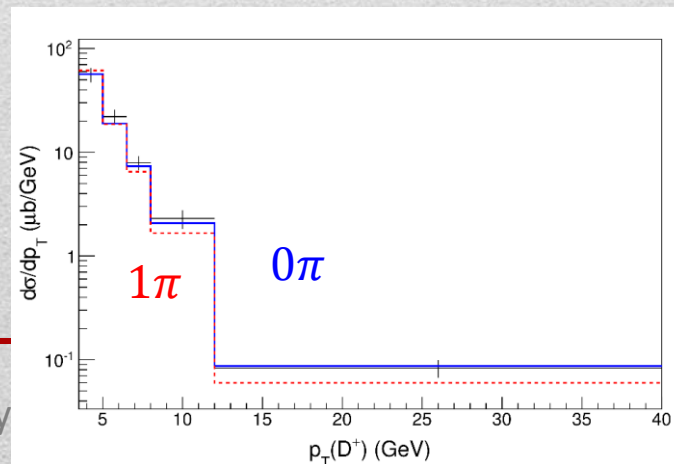
This picture could spoil existing meson distributions used to tune MC
We verify this is not the case up to an overall K factor

Guerrieri, Piccinini, AP, Polosa, PRD90, 034003

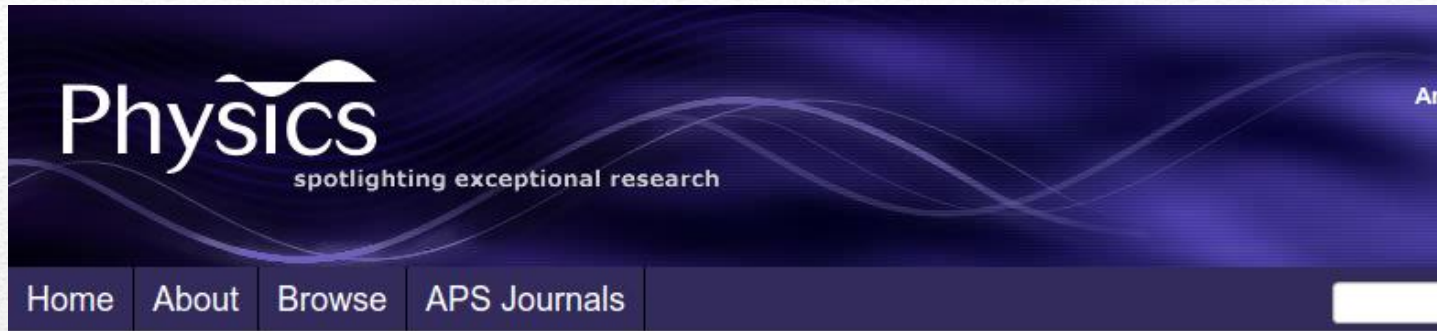
Neither at CDF...



...nor at ATLAS



$$Z_c(3900)$$



Notes from the Editors: Highlights of the Year

Published December 30, 2013 | *Physics* 6, 139 (2013) | DOI: 10.1103/Physics.6.139

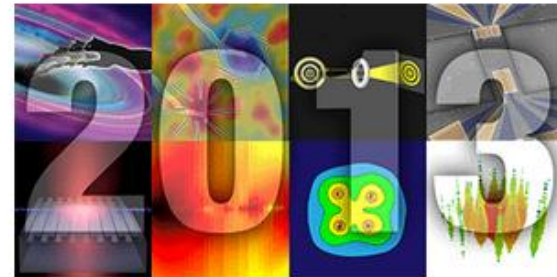
***Physics* looks back at the standout stories of 2013.**

As 2013 draws to a close, we look back on the research covered in *Physics* that really made waves in and beyond the physics community. In thinking about which stories to highlight, we considered a combination of factors: popularity on the website, a clear element of surprise or discovery, or signs that the work could lead to better technology. On behalf of the *Physics* staff, we wish everyone an excellent New Year.

— Matteo Rini and Jessica Thomas

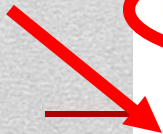
Four-Quark Matter

Quarks come in twos and threes—or so nearly every experiment has told us. This summer, the BESIII Collaboration in China and the Belle Collaboration in Japan reported they had sorted through the debris of high-energy electron-positron collisions and seen a **mysterious particle** that appeared to contain four quarks. Though other explanations for the nature of the particle, dubbed $Z_c(3900)$, are possible, the “tetraquark” interpretation may be gaining traction: BESIII has since **seen** a series of other particles that appear to contain four quarks.



Images from popular *Physics* stories in 2013.

mysterious particle

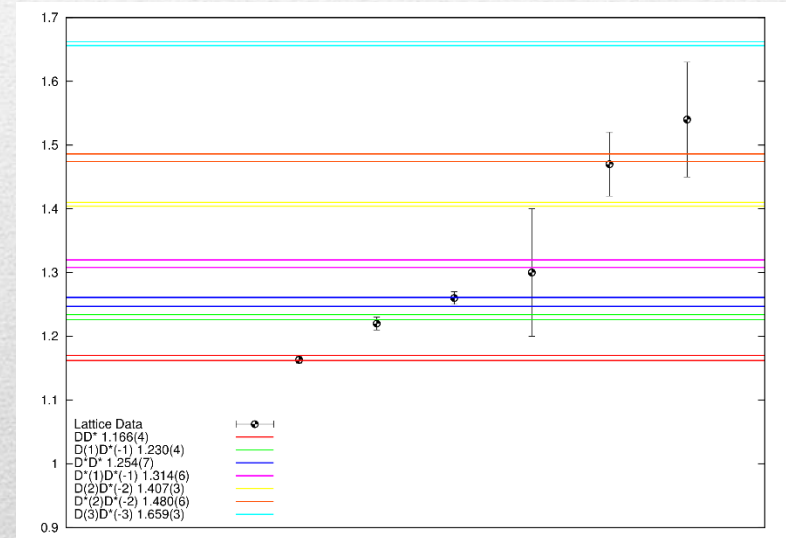
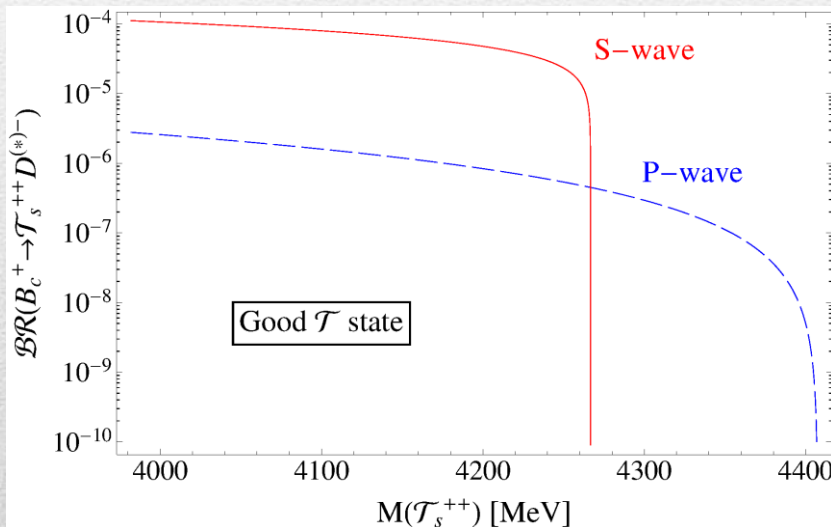


Doubly charmed states

For example, we proposed to look for **doubly charmed states**, which in tetraquark model are $[cc]_{S=1}[\bar{q}\bar{q}]_{S=0,1}$

These states could be observed in B_c decays @LHC and sought on the lattice

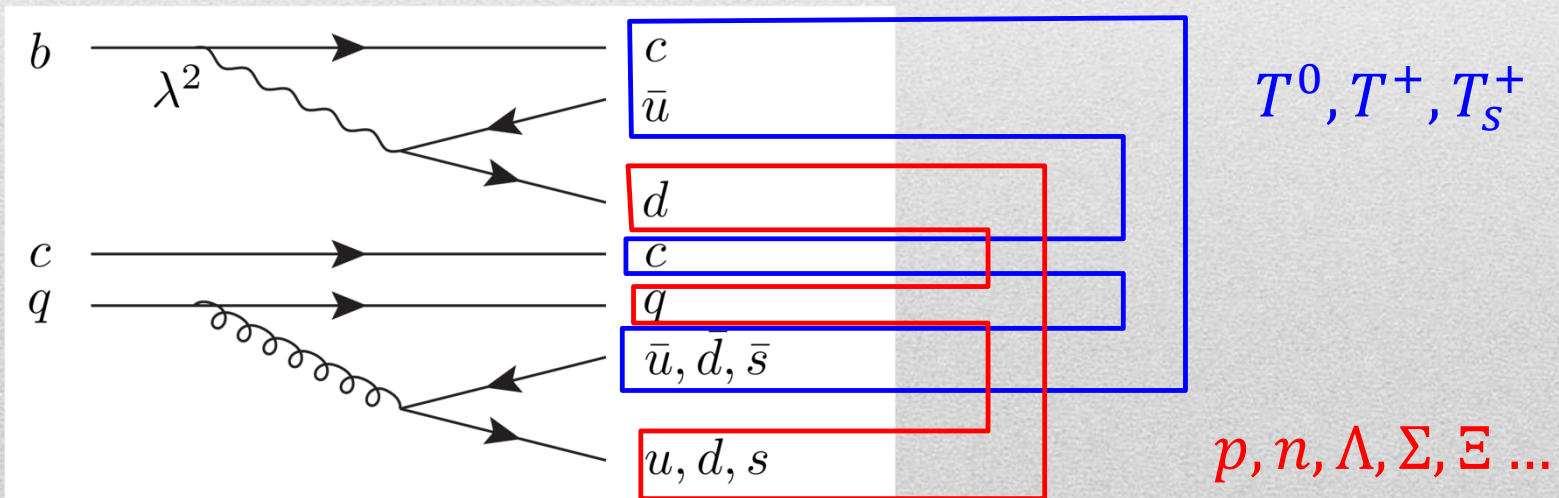
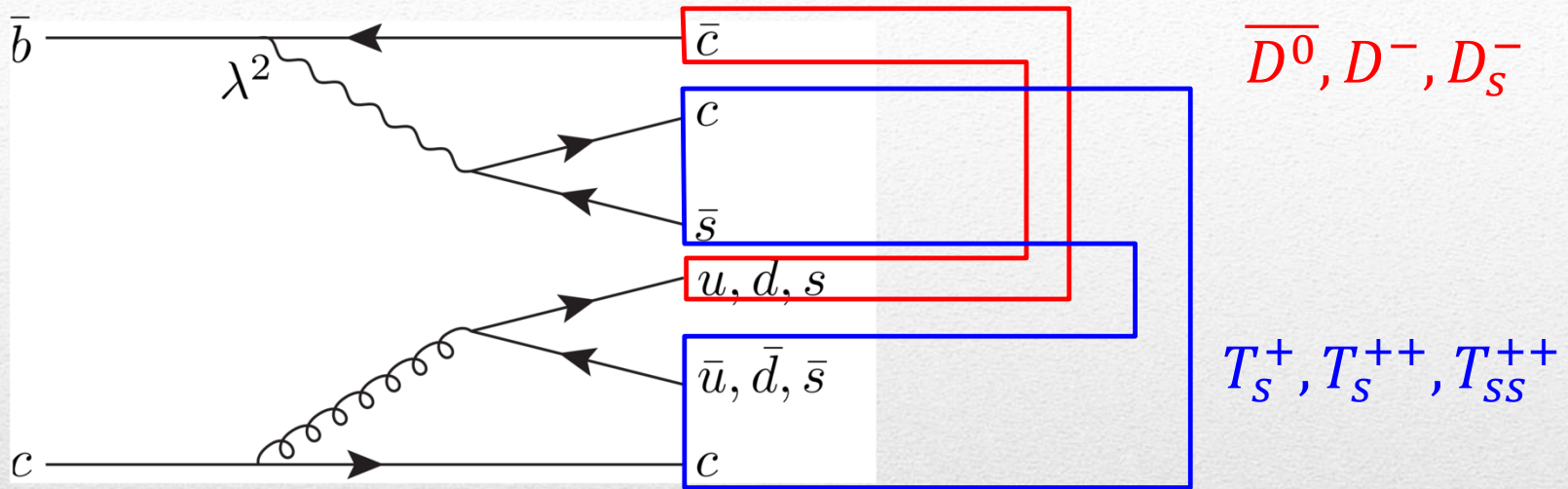
Esposito, Papinutto, AP, Polosa, Tantalò, PRD88 (2013) 054029



Preliminary results on spectrum for $m_\pi = 490$ MeV, $32^3 \times 64$ lattice, $a = 0.075$ fm

Guerrieri, Papinutto, AP, Polosa, Tantalò, PoS LATTICE2014 106

T states production



Prompt production of $X(3872)$

$X(3872)$ is the Queen of exotic resonances, the most popular interpretation is a $D^0\bar{D}^{0*}$ molecule (bound state, pole in the 1st Riemann sheet?)

We aim to evaluate prompt production cross section at hadron colliders via Monte-Carlo simulations

Q. What is a molecule in MC? **A.** «Coalescence» model



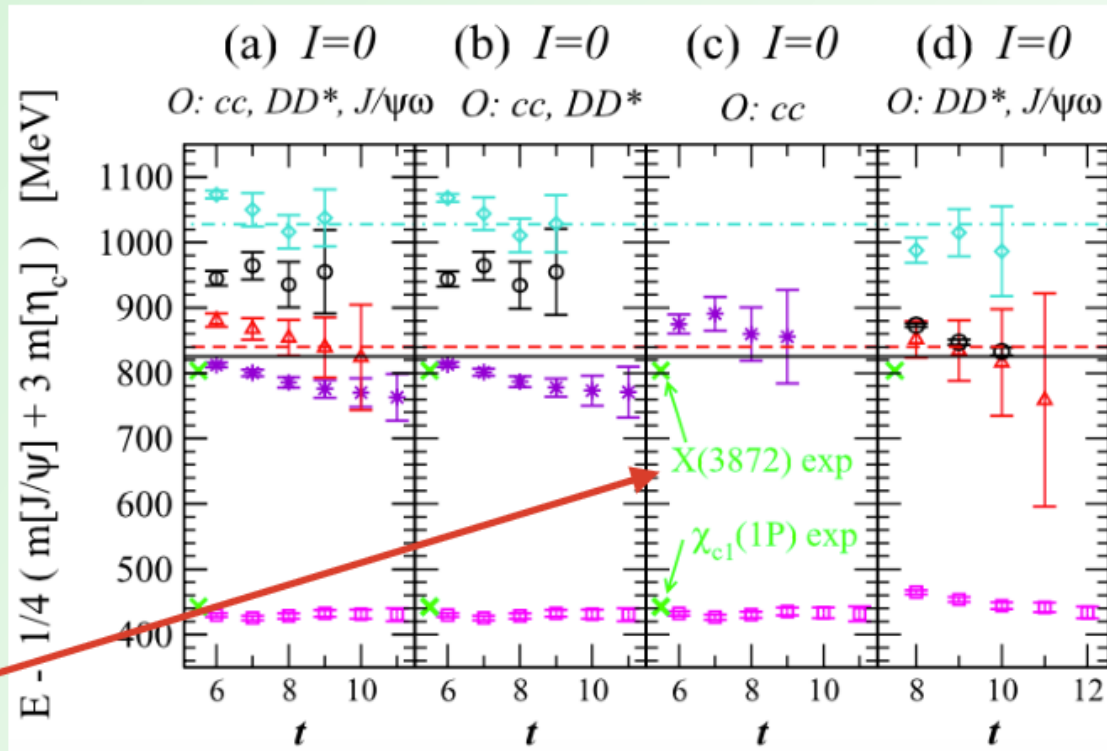
$$\sigma(p\bar{p} \rightarrow X(3872)) \sim \int d^3k |\langle X | D\bar{D}^* \rangle \langle D\bar{D}^* | p\bar{p} \rangle|^2 < \int_{k < k_{max}} d^3k |\langle D\bar{D}^* | p\bar{p} \rangle|^2$$

This should provide an upper bound for the cross section

Bignamini, Piccinini, Polosa, Sabelli PRL103 (2009) 162001

Kadastic, Raidan, Strumia PLB683 (2010) 248

X(3872) on the lattice: spectrum



Caveats:

- Small lattices, large artifacts
- Three body dynamics may play a role
- Interpretation of the overlap coefficients is questionable

Status of other XYZ on the lattice is even less clear

Where is the $\chi_{c1}(2P)$?

Prelovsek *et al.* PRL 111 (2013) 192001 arXiv: 1307.5172

Estimating k_{max}

The binding energy is $E_B \approx -0.16 \pm 0.31$ MeV: **very small!**

In a simple square well model this corresponds to:

$$\sqrt{\langle k^2 \rangle} \approx 50 \text{ MeV}, \sqrt{\langle r^2 \rangle} \approx 10 \text{ fm}$$

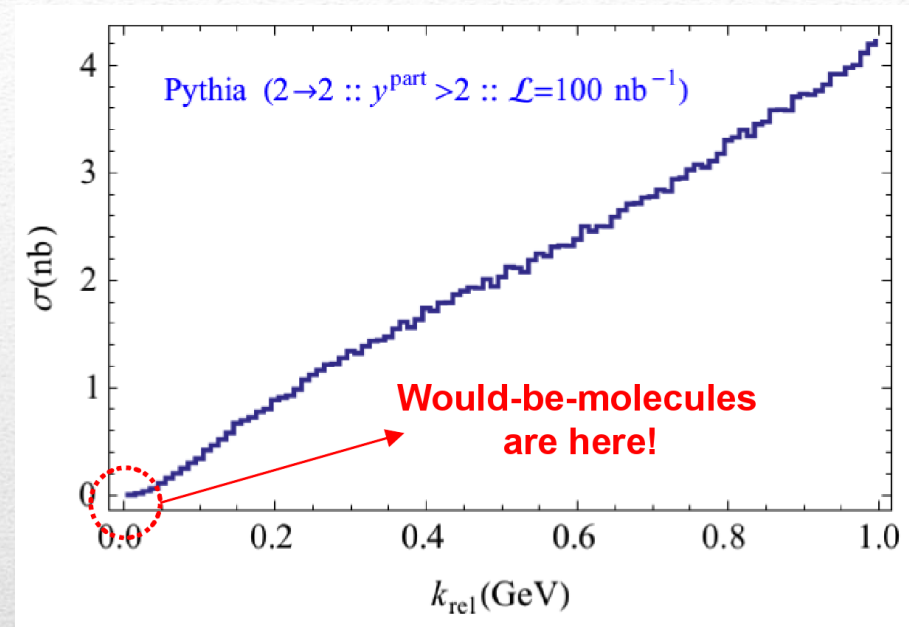
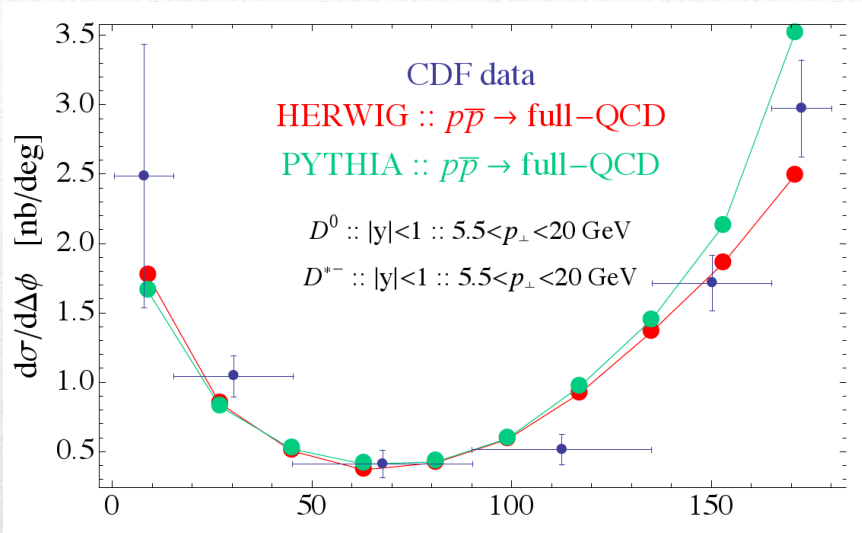
$$\left(\begin{array}{l} \text{binding energy reported in Kamal Seth's talk is } E_B \approx -0.013 \pm 0.192 \text{ MeV:} \\ \sqrt{\langle k^2 \rangle} \approx 30 \text{ MeV}, \sqrt{\langle r^2 \rangle} \approx 30 \text{ fm} \end{array} \right)$$

to compare with deuteron: $E_B = -2.2$ MeV

$$\sqrt{\langle k^2 \rangle} \approx 80 \text{ MeV}, \sqrt{\langle r^2 \rangle} \approx 4 \text{ fm}$$

We assume $k_{max} \sim \sqrt{\langle k^2 \rangle} \approx 50$ MeV, some other choices are commented later

2009 results

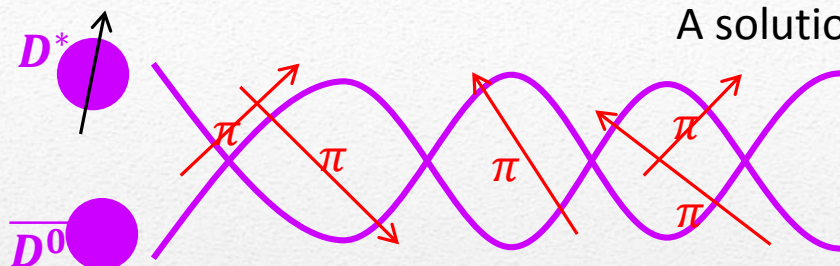


We tune our MC to reproduce CDF distribution of $\frac{d\sigma}{d\Delta\phi} (p\bar{p} \rightarrow D^0 D^{*-})$

We get $\sigma(p\bar{p} \rightarrow DD^* | k < k_{max}) \approx 0.1 \text{ nb}$ @ $\sqrt{s} = 1.96 \text{ TeV}$

Experimentally $\sigma(p\bar{p} \rightarrow X(3872)) \approx 30 - 70 \text{ nb}!!!$

Estimating k_{max}



A solution can be FSI (rescattering of DD^*), which allow k_{max} to be as large as $5m_\pi \sim 700$ MeV

$$\sigma(p\bar{p} \rightarrow DD^* | k < k_{max}) \approx 230 \text{ nb}$$

Artoisenet and Braaten, PRD81, 114018

However, the applicability of Watson theorem is challenged by the presence of pions that interfere with DD^* propagation

Bignamini, Grinstein, Piccinini, Polosa, Riquer, Sabelli, PLB684, 228-230

FSI saturate unitarity bound? Influence of pions small?

Artoisenet and Braaten, PRD83, 014019

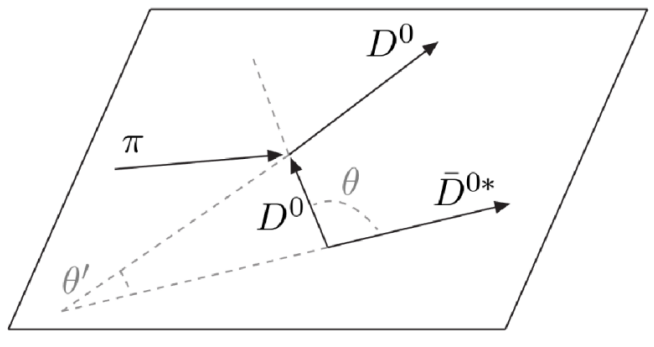
Guo, Meissner, Wang, Yang, JHEP 1405, 138; EPJC74 9, 3063; CTP 61 354
use $E_{max} = M_X + \Gamma_X$ for above-threshold unstable states

With different choices, 2 orders of magnitude uncertainty,
limits on predictive power

A new mechanism?

In a more **billiard-like** point of view, the comoving pions can **elastically interact** with $D(D^*)$, and **slow down** the pairs DD^*

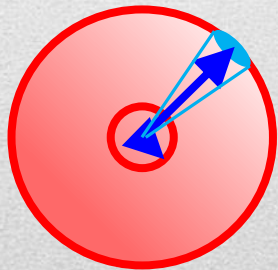
Esposito, Piccinini, AP, Polosa, JMP 4, 1569
Guerrieri, Piccinini, AP, Polosa, PRD90, 034003



The mechanism also implies: D mesons actually **“pushed”** **inside** the potential well (the **classical 3-body problem!**)

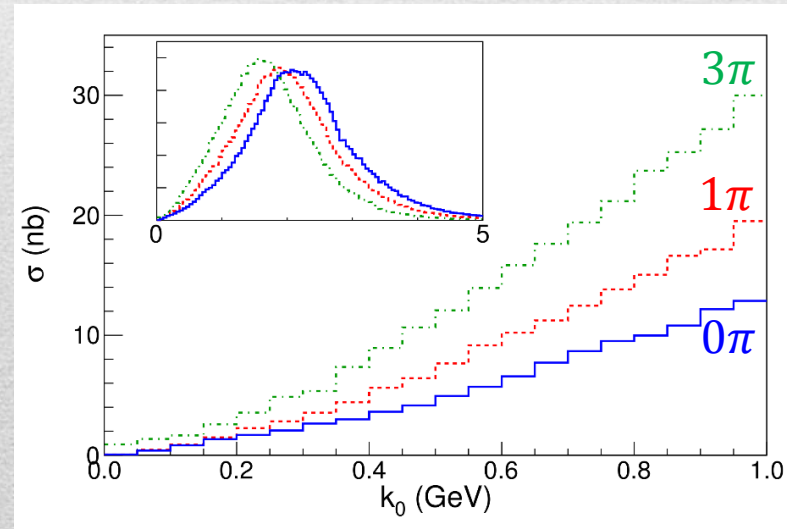
$X(3872)$ is a **real, negative energy bound state** (stable)

It also explains a small width $\Gamma_X \sim \Gamma_{D^*} \sim 100$ keV



By comparing hadronization times of heavy and light mesons, we estimate up to ~ 3 collisions can occur before the heavy pair to fly apart

We get $\sigma(p\bar{p} \rightarrow X(3872)) \sim 5$ nb, **still not sufficient** to explain all the experimental cross section



Hybridized tetraquarks – Selection rules

- Consider the **down quark part of the $X(3872)$** in the diquarkonium picture:

$$\Psi_{\mathbf{d}} = X_{\mathbf{d}} = [cd]_0[\bar{c}\bar{d}]_1 + [cd]_1[\bar{c}\bar{d}]_0 \sim (D^{*-}D^+ - D^{*+}D^-) + i(\psi \times \rho^0 - \psi \times \omega^0)$$

↑
Fierz rearrangement

- The closest threshold from below is $\Psi_m \sim \bar{D}^0 D^{*0} \longrightarrow \Psi_{\mathbf{d}} \perp \Psi_m$ ✓

- But if we consider the **up quark part of the $X(3872)$** :

$$\Psi_{\mathbf{d}} = X_{\mathbf{u}} = [cu]_0[\bar{c}\bar{u}]_1 + [cu]_1[\bar{c}\bar{u}]_0 \sim (\bar{D}^{*0}D^0 - D^{*0}\bar{D}^0) - i(\psi \times \rho^0 + \psi \times \omega^0)$$

- But then $\longrightarrow \Psi_{\mathbf{d}} \not\perp \Psi_m$ ✗

- Only $X_{\mathbf{d}}$ is produced via this mechanism
 - isospin violation
 - no hyperfine neutral doublet

- X_b (A) Diquark model predicts $M(X_b) \simeq M(Z_b) \simeq (10607 \pm 2)$ MeV
 (B) The closest orthogonal threshold is $M(B^0 B^{*0}) = (10604.4 \pm 0.3)$ MeV
 (C) This could either be **above threshold (very narrow state)** or **below (no state at all)**
 (D) Experimentally the diquark model overpredicts the mass of the X :

$$M(Z_c) - M(X) \simeq 32 \text{ MeV}$$

- (E) We **favor the below threshold scenario** \longrightarrow **no X_b should be seen**


A. Esposito

Production of hybridized tetraquarks

Going back to $pp(\bar{p})$ collisions, we can imagine hadronization to produce a state

$$|\psi\rangle = \alpha|[qQ][\bar{q}\bar{Q}]\rangle_c + \beta|(\bar{q}q)(\bar{Q}Q)\rangle_o + \gamma|(\bar{q}Q)(\bar{Q}q)\rangle_o$$

If $\beta, \gamma \gg \alpha$, an initial tetraquark state is not likely to be produced
The open channel mesons fly apart
(see MC simulations)



If hybridization mechanism is at work, an open state can resonate in a closed one

α expected to be small in Large N limit, [Maiani, Polosa, Riquer JHEP 1606, 160](#)

No prompt production without hybridization mechanism!

Note that only the $X(3872)$ has been observed promptly so far...

...and a narrow $X(4140)$ not compatible with the LHCb one → **needs confirmation**