Written test of Advanced Quantum Mechanics

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Exam time: 2 hours. You can use the Clebsch-Gordan sheet by PDG.

EXERCISE 1

A spin 1/2 particle constrained to move on a sphere has a wave function

$$\psi = A \sin\theta \cos\phi \left(3\chi_+ + 4e^{i\alpha}\chi_-\right) \tag{1}$$

where A is a normalization constant, α is a constant phase, $\chi_\pm$ are the eigenstates of S_z with eigenvalues $\pm\hbar/2$, respectively.

(a) If a measurement of J^2, where $\mathbf{J} = \mathbf{L} + \mathbf{S}$, is performed on ψ, what values can be obtained and with what probability?

(b) A measurement of L_z is performed on ψ, obtaining $\hbar$, and immediately after, a measurement of J_z obtaining $\hbar/2$. Write the normalized wave function ψ_1 of the system after the two measurements.

(c) The system with wave function ψ_1 evolves with Hamiltonian

$$H = \frac{\omega}{\hbar} J^2. \tag{2}$$

Calculate the measurable values of L_z at time t and their respective probabilities.

EXERCISE 2

Consider two identical particles with spin 1/2 and mass m that interact in the center-of-mass system with Hamiltonian

$$H_0 = \frac{p^2}{m} + \frac{1}{4} m\omega^2 r^2 + 2\frac{\alpha}{\hbar} \mathbf{L} \cdot \mathbf{S} \tag{3}$$

where r is the relative position, p the corresponding conjugate momentum, $\mathbf{L}$ the orbital angular momentum, and $\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$ the total spin. Assume $\alpha \ll \omega$.

Calculate the energy of the ground state and the first three excited state and their degeneracies.